*บทความวิจัย*

# ความอลวนที่เกิดขึ้นในเวลาเดียวกันระหว่างระบบอลวนที่ต่างกันโดยใช้ตัวควบคุมแบบแอคทีฟ

**ธงชัย บทมาตย์***

**บทคัดย่อ**

งานวิจัยนี้ได้ศึกษา ความอลวนที่เกิดขึ้นในเวลาเดียวกันระหว่างระบบอลวนที่ต่างกันโดยใช้ตัวควบคุมแบบแอคทีฟ ขั้นตอนวิธีนี้ได้ถูกประยุกต์ใช้กับความอลวนที่เกิดขึ้นในเวลาเดียวกันของระบบพลศาสตร์แบบ Lü และ Chen ประสิทธิภาพของตัวควบคุมได้ถูกแสดงผลแบบเชิงตัวเลข

**คำสำคัญ:** ความอลวน ตัวควบคุมแบบแอคทีฟ การเกิดขึ้นในเวลาเดียวกัน

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# Chaos Synchronization Between Two Different Hyperchaotic Systems Using Active Control

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## ABSTRACT

This paper presents chaos synchronization between two different hyperchaotic systems by using active control. This technique is applied to achieve chaos synchronization of the dynamical systems Lü and Chen. The performances of the control schemes are verified by numerical simulations.

**Keyword**: hyperchaotic, active control, synchronization

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## Introduction

Since Pecora and Carrol introduced a method [1] to synchronization two identical systems with different initial conditions, chaos and hyperchaotic synchronization , as a very important topic in the nonlinear science, has been developed extensively in the last few years. A wide variety of approaches has been proposed for the synchronization of hyperchaotic systems which include linear and nonlinear feedback control [2-4], active control [5]. Most of the method mentioned above synchronization is two identical hyperchaotic systems. However, the method of the synchronization of two different hyperchaotic systems is far from being straight forward. Hyperchaotic consists of different structures and parameters mismatch of the two hyperchaotic systems. It is well known that, to generate hyperchaos from the dissipatively autonomously polynomial systems, the state equation must satisfy the following two basic conditions. Firstly, the dimension of the state equation is at least 4 and the order of the state equation is at least 2. Secondly, the systems has at least two positive Lyapunov exponents satisfying that the sum of all Lyapunov exponents is less than zero.

In this paper, we apply active control theory to synchronize two different hyperchaotic systems. We demonstrate active control technique by Lü and Chen systems.

## Preliminary

### Definition 1

We consider the system described by

$$\dot{x} = f(t, x) \tag{1}$$

where $x \in R^n$, $\dot{x} = \left[\frac{dx_1}{dt}, \frac{dx_2}{dt}, \cdots, \frac{dx_n}{dt}\right]$ and is a vector having components $f_i\ (x_1, x_2, \ldots, x_n)$, $i = 1, 2, \ldots, n$.

We shall assume that the $f_i$ are continuous and satisfy standard conditions, such as having continuous first partial derivatives so that the solution of (1) exists and is unique for given initial conditions. If $f_i$ do not depend explicitly on t, (1) is called autonomous. If $f(c, t) = 0$ for all t, where c is some constant vector, then it follows at once from (1) that if $x(t_0) = c$ then $x(t) = c$ for all $t \geq t_0$. Thus solutions starting at c remain there, and c is said to be an equilibrium or critical point.

An equilibrium state $x = 0$ is said to be

1. **Stable** if for any positive scalar $\varepsilon$ there exists a positive scalar $\delta$ such that $\| x(t_0)\|_e < \delta$ implies $\| x(t)\|_e < \varepsilon$, $t \geq t_0$, $\|.\|_e$ is a standard Eucledian norm.
2. **Asymptotically stable** if it stable and if in addition $x(t) \longrightarrow 0$ as $t \longrightarrow \infty$.
3. **Unstable** if it is not stable; that is, there exists an $\varepsilon > 0$ such that for every $\delta > 0$ there exist an $x(t_0)$ with $\| x(t_0)\|_e < \delta$ so that $\| x(t_1)\|_e \geq \varepsilon$ for some $t_1 > t_0$. If this holds for every $x(t_0)$ in $\| x(t_0)\|_e < \delta$ the equilibrium is completely unstable.

**Definition 2 Algebraic criteria for linear systems**

Before studying nonlinear systems we return to the general continuous time linear system.

$$\dot{x} = Ax \tag{2}$$

where A is constant n x n matrix, and (2) may represent the closed or opened loop system. Provided det A $\neq 0$, the only equilibrium point of (2) is the origin, so it is meaningful to refer to the stability of the system (2). The two basic results on which the development of linear system stability theory relies are now given.

**Theorem 1.** The system (2) is asymptotically stable if and only if A is a stability matrix, i.e. all the characteristic roots $\lambda_k$ of A have negative real parts; (2) is unstab le if for some characteristic roots $\lambda_k$, $Re(\lambda_k) > 0$; and completely unstable if for all characteristic roots $\lambda_k$, $Re(\lambda_k) > 0$

To begin with, the definition of hyperchaotic synchronization used in this paper is given below.

**Definition 3 [8]. For two nonlinear hyperchaotic systems**

$$\dot{x} = f(t, x) \tag{3}$$

$$\dot{y} = g(t, y) + u(t, x, y) \tag{4}$$

where $x, y \in R^n$, $f, g \in C^r [R^+ \times R^n, R^n]$, $u \in C^r [R^+ \times R^n \times R^n, R^n]$, $r \geq 1$ assume that (3) is the drive system, and (4) is the response system, u (t, x, y) is the control vector. Response system and drive system are said to be synchronic if for $\forall x(t_0)$, $y(t_0) \in R^n$, $\lim_{t\to\infty} \|x(t) - y(t)\| = 0$.

**Hyperchaotic synchronization between hyperchaotic Chen and hyperchaotic Lü**

The hyperchaotic Chen system [7] , is given by

$$\dot{x} = a(y-x) + w$$
$$\dot{y} = dx - xz + cy \qquad (5)$$
$$\dot{z} = xy - bz$$
$$\dot{w} = yz + rw$$

where $(a, b, c, d, r) \in R$. When $a = 35$, $b = 3$, $c = 12$, $d = 7$, $0.085 < r \leq 0.798$, system (5) is hyperchaotic.

The hyperchaotic Lü system [6], is given by

$$\dot{x} = a_1 (y-x)$$
$$\dot{y} = -xy + c_1 y + w \qquad (6)$$
$$\dot{z} = xy - b_1 z$$
$$\dot{w} = z - d_1 w$$

where $(a_1, b_1, c_1, d_1) \in R$. When $a_1 = 15$, $b_1 = 5$, $c_1 = 10$, $d_1 = 1$, system (6) is hyperchaotic.

In particular, these projections of chaotic attractor of hyperchaotic Chen system (5) with $a = 35$, $b = 3$, $c = 12$, $d = 7$, $r = 0.5$ are displayed in Fig. 1 and hyperchaotic Lü systems (6) with $a_1 = 15$, $b_1 = 5$, $c_1 = 10$, $d_1 = 1$ are displayed Fig. 2.

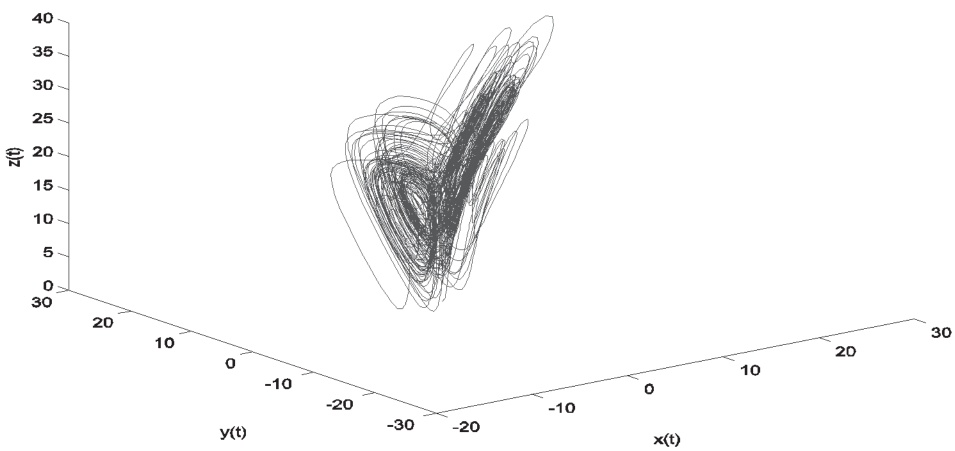


**Figure 1** Hyperchaotic Chen chaotic attractor.

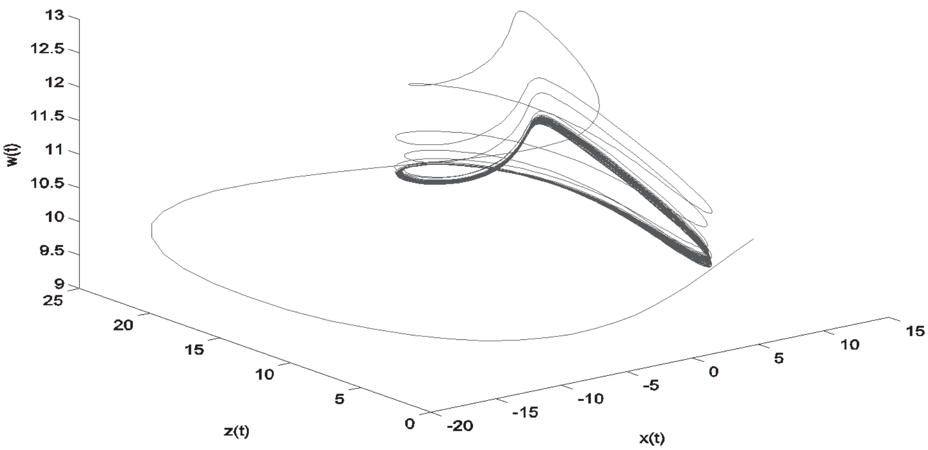


**Figure** 2 Hyperchaotic Lü chaotic attractor.

Our aim is to make hyperchaotic synchronization between hyperchaotic Chen system and hyperchaotic Lü system by using active control. We assume that hyperchaotic Chen system is the drive system and hyperchaotic Lü system is the response system. Therefore, the drive system is defined as follows

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + w_1 \\ \dot{y}_1 &= dx_1 - x_1 z_1 + cy_1 \\ \dot{z}_1 &= x_1 y_1 - bz_1 \\ \dot{w}_1 &= y_1 z_1 + rw_1 \end{aligned} \tag{7}$$

and the response system is given by

$$\begin{aligned} \dot{x}_2 &= a_1(y_2 - x_2) + u_1(t) \\ \dot{y}_2 &= x_2 z_2 + c_1 y_2 + w_2 + u_2(t) \\ \dot{z}_2 &= x_2 y_2 - b_1 z_2 + u_3(t) \\ \dot{w}_2 &= z_2 - d_1 w_2 + u_4(t). \end{aligned} \tag{8}$$

We have introduced four control functions $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ in (8). Our goal is to determine the control functions $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$. In order to estimate the control functions, we subtract (7) from (8). We define the error system as the difference between system (7) and the controlled system (8). Let us define the state error between the response system (8) that is to be controlled and the controlling system (7) as

$$e_1 = x_2 - x_1,\ e_2 = y_2 - y_1,\ e_3 = z_2 - z_1,\ e_4 = w_2 - w_1. \quad (9)$$

Subtracting (7) from (8) and using the notation in (9) yields

$$\begin{aligned} \dot{e}_1 &= -w_1 + a_1(e_2-e_1) + (a_1-a)(y_1-x_1) + u_1(t) \\ \dot{e}_2 &= -e_1e_3 - x_1e_3 - z_1e_1 + c_1e_2 + (c_1-c)y_1 - dx_1 + w_2 + u_2(t) \quad (10) \\ \dot{e}_3 &= e_1e_2 + y_1e_1 + x_1e_2 - re_3 - (b-b_1)z_1 + u_3(t) \\ \dot{e}_4 &= z_2(1-y_1) + y_1e_3 - d_1e_4 - (d_1+r)w_1 + u_4(t). \end{aligned}$$

We define the active control functions $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ as follows

$$\begin{aligned} u_1(t) &= v_1(t) - (a_1-a)(y_1-x_1) + w_1 \\ u_2(t) &= v_2(t) + e_1e_3 + x_1e_3 + z_1e_1 - (c_1-c)y_1 - w_2 + dx_1 \quad (11) \\ u_3(t) &= v_3(t) - e_1e_2 - y_1e_1 - x_1e_2 + (b-b_1)z_1 \\ u_4(t) &= v_4(t) - z_2(1-y_1) - y_1e_3 + (d_1+r)w_1 \quad . \end{aligned}$$

Hence the error system (10)

$$\begin{aligned} \dot{e}_1 &= a_1(e_2-e_1) + v_1(t) \\ \dot{e}_2 &= c_1e_2 + v_2(t) \\ \dot{e}_3 &= -b_1e_3 + v_3(t) \\ \dot{e}_4 &= d_1e_4 + v_3(t). \quad (12) \end{aligned}$$

The error system (12) to be controlled is a linear system with a control input $v_1(t)$, $v_2(t)$, $v_3(t)$ and $v_4(t)$ as function of the error state $e_1$, $e_2$, $e_3$, and $e_4$. As long as these feedback the system $e_1$, $e_2$, $e_3$ and $e_4$ converges to zero as time t tends to infinity. This implies that hyperchaotic Chen and hyperchaotic Lü system are synchronized with feedback control. These are many possible choices for the control $v_1(t)$, $v_2(t)$, $v_3(t)$ and $v_4(t)$. We choose

$$\begin{bmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \\ V_4(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix},$$

where A is a $4 \times 4$ constant matrix. In order to make the closed loop system stable, the proper choice of the elements of the matrix A is such that feedback system must have all eigenvalues with negative real parts. The matrix A is chosen in the following form.

$$A = \begin{bmatrix} -1+a_1 & -a & 0 & 0 \\ 0 & -(1+c_1) & 0 & 0 \\ 0 & 0 & -1+b_1 & 0 \\ 0 & 0 & 0 & -1+d_1 \end{bmatrix}.$$

In this particular choice, the closed loop system (12) has the eigenvalues -1, -1, -1 and -1. This choice will lead to the error states $e_1$, $e_2$, $e_3$ and $e_4$ converge to zero as time tends to infinity and hence the synchronization between hyperchaotic Chen and hyperchaotic Lü is achieved.

## Numerical simulations

Fourth-order Runge-Kutta integration method is used to solve two systems of differential equations (7) and (8) with time step size 0.001. We select the parameters of hyperchaotic Chen system as $a = 35$, $b = 3$, $c = 12$, $d = 7$, $r = 0.5$ and the parameters of hyperchaotic Lü system as $a_1 = 15$, $b_1 = 5$, $c_1 = 10$ and $d_1 = 1$, so that each of hyperchaotic Chen system and hyperchaotic Lü system exhibits a chaotic behavior. The initial values of the drive system are $x_1(0) = 0.5$, $y_1(0) = 1$, $z_1(0) = 1.5$, $w_1(0) = 2.5$ and the initial values of the response system are $x_2(0) = 10$, $y_2(0) = -5$, $z_2(0) = 5$, $w_2(0) = -10$. Then the initial values of the error system are $e_1(0) = 9.5$, $e_2(0) = -6$, $e_3(0) = 3.5$, $e_4(0) = 7.5$.

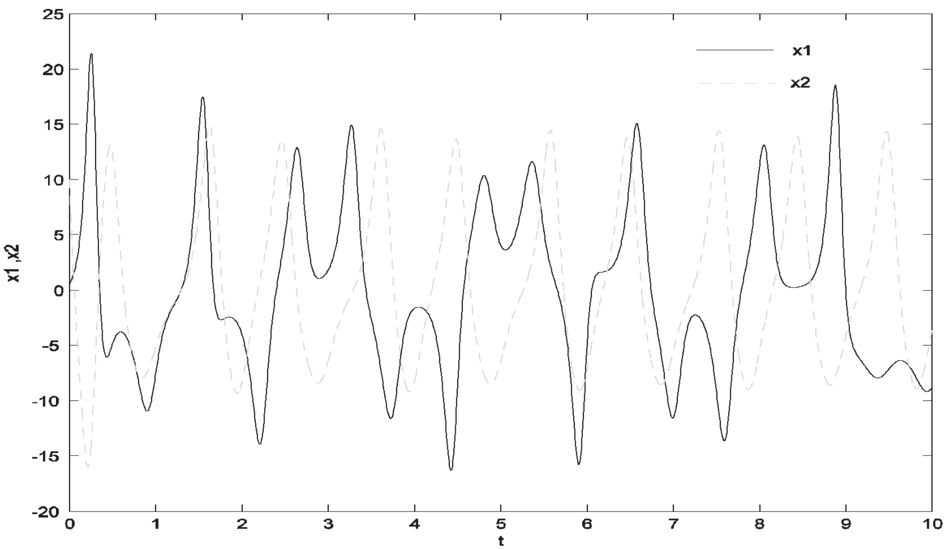


**Figure** 3 The state $x_1$ of hyperchaotic Chen system and hyperchaotic Lü system without active control.

The results of the simulation of hyperchaotic Chen system and hyperchaotic Lü system without active control are shown in Fig. 3 (displays $x_1$ and $x_2$) and Fig. 4 (displays $e_1$, $e_2$, $e_3$, and $e_4$) . Fig. 5 shows that the synchronization is occurred after applying active control. Fig. 6 shows the state errors of hyperchaotic Chen system and hyperchaotic Lü system of equations with the active control activated.

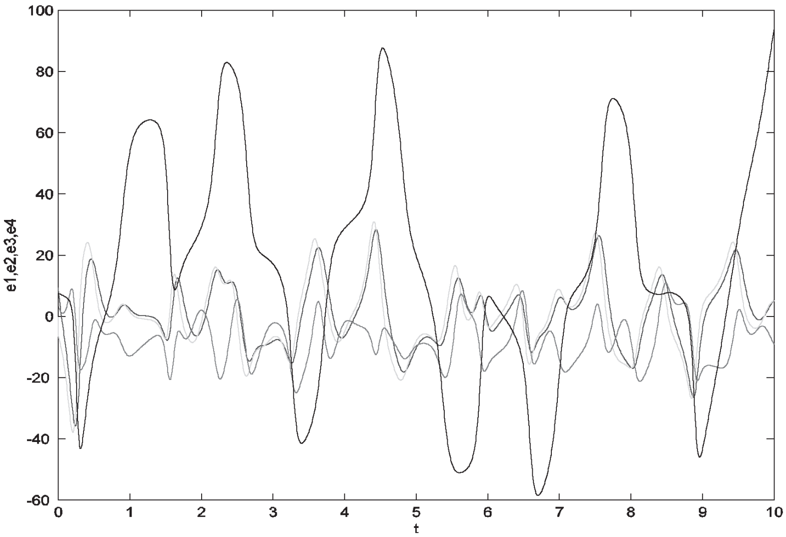


**Figure 4** The state errors ($e_1$, $e_2$, $e_3$, $e_4$) of hyperchaotic Chen system and hyperchaotic Lü system of equations without the active control.

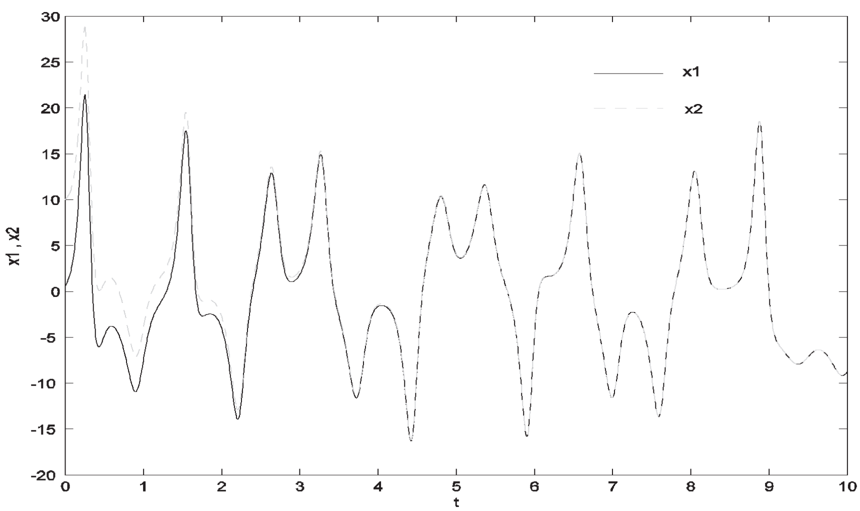


**Figure 5** The state $x_1$ of hyperchaotic Chen system and $x_2$ hyperchaotic Lü system with active control activated.

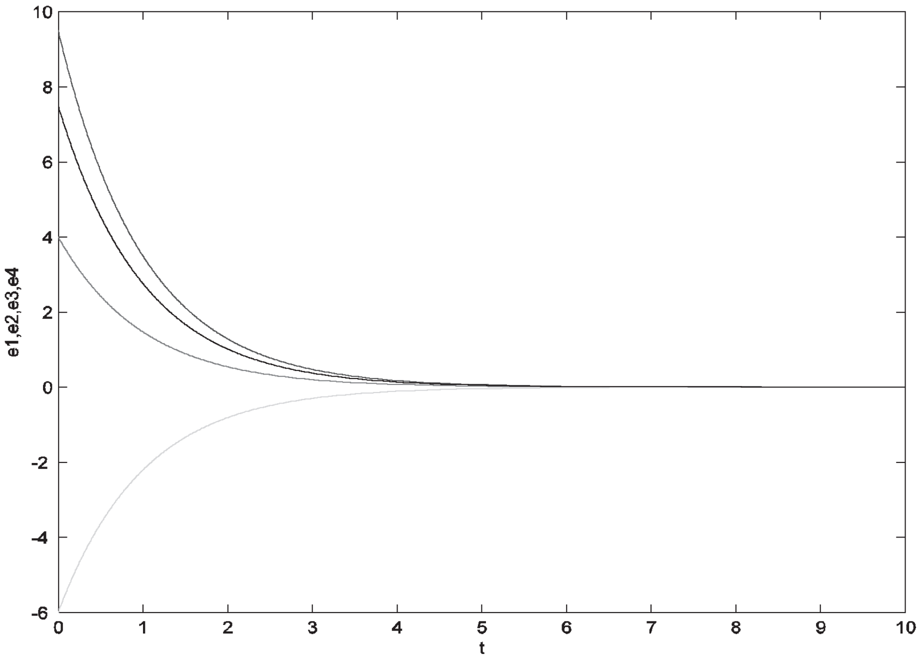


**Figure 6** The state errors $e_1$, $e_2$, $e_3$, $e_4$ of hyperchaotic Chen system and hyperchaotic Lü system of equations with the active control activated.

## Conclusion

This work demonstrates the chaos synchronization between two different hyperchaotic systems using active control achieved. The hyperchaotic Lü system is controlled to be hyperchaotic Chen system. We can use active control theory to synchronize two identical or different hyperchaotic systems.

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