

แบบจำลองเชิงพลศาสตร์ของการกระโดดของอิเล็กตรอน ในวงแหวนควอนตัมคู่

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บทคัดย่อ

แบบจำลองของอิเล็กตรอนที่เคลื่อนที่ผ่านวงแหวนควอนตัมภายใต้สนามแม่เหล็กคงตัว สามารถจำลองได้โดยการสร้างตัวแปลงเพื่อแก้ชุดสมการเชิงอนุพันธ์ให้ไปอยู่ในรูปคาโนนิคอลซึ่งแรงลอเรนซ์ที่เกิดขึ้นทำให้อิเล็กตรอนที่เคลื่อนที่ผ่านวงแหวนมีการเอนเอียงไปด้านหนึ่ง และขณะเดียวกันผลจากปรากฏการณ์ของ Aharonov-Bohm ซึ่งเป็นการเลื่อนเฟสได้ทำให้อิเล็กตรอนที่ปลายวงแหวนเกิดการแทรกสอด ทำให้ความสามารถในการส่งผ่านลดลง ผลลัพธ์จากการคำนวณแสดงให้เห็นว่าสำหรับกรณีที่ไม่มีสนามแม่เหล็กหรือสนามแม่เหล็กที่เกิดขึ้นบนวงแหวนทั้งสองมีค่าใกล้เคียงกัน อิเล็กตรอนจะสามารถเคลื่อนที่ผ่านวงแหวนได้ แต่สำหรับกรณีที่สนามแม่เหล็กมีค่าแตกต่างกันมาก อิเล็กตรอนจะไม่สามารถเคลื่อนที่ผ่านวงแหวนวงแรกได้

คำสำคัญ: ควอนตัมดอท วงแหวนควอนตัม

Dynamical Simulation of Electron Hopping in Double Quantum Rings

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ABSTRACT

The wave packet of electron hopping through quantum rings under the static magnetic field is solved by transformation to the canonical form of certain system of differential equations. The Lorentz force leads to electron asymmetry which enhances the electron passing through a quantum ring while the Aharonov–Bohm effect (AB effect) reduces the probability of transmission by phase shifted interference. For zero or similar magnetic field of both rings, the wave packet can pass both quantum rings to the exit quantum wire while different magnetic field of both rings prevent the second ring's injection of electron.

Keywords: quantum ring, quantum dot

Introduction

The quantum dot is a portion of matter that confined electron in spatial three dimensions. Sorting of many quantum dots give new material structure such as quantum wire. The quantum rings is produced by quantum dots in two dimensions circle pattern. The electrons are hopping through between quantum dots that generate the current in system. The wave function of an electron passing the quantum rings under magnetic field can be controlled the arms of passing. In addition the Aharonov–Bohm (AB) effect give the explanation of occurring interference by phase shift of both arms [1]. Oscillations of the electric properties were detected in metal [2]. The theory the AB oscillations in one–dimension was developed which the magnetic field is inaccessible for electrons and affect to the vector potential and the scattering matrix assumed transport symmetry with respect to ring [3]. But this assumption ignores the Lorentz force effect that is not exactly true for semiconductor. The effect of Lorentz force for quantum system was discussed for interference pattern form an electron traveling in the magnetic field [4]. And the effect of the magnetic field on the electron in quantum rings was previously discussed [5].

This paper presents the result of two quantum rings that are connected and separated by some quantum dots for protecting the transmission over affecting quantum dots between rings. The model and parameters are following on [6], which is observed in GaAs quantum rings [7]. The approach of the incoming electron is defined by time–dependent approach and the initial calculation is performed in a basis of Gaussian functions. The two rings are separated by a few quantum dots for protecting to effective between many dots between rings.

Methods

The mathematical model is an electron confined in quantum dots system on a line before being transverse through the first quantum ring. The magnetic field is perpendicular to the x-y plane along z-direction. The local wave function in quantum dots is assumed to be Gaussian wave functions,

$$f_n(x,y) = e^{-\frac{1}{\lambda\sqrt{\pi}}\left(\frac{(\vec{r}-\vec{R}_n)^2}{2\lambda^2} + \frac{ieB(x-X_n)(y+Y_n)}{2\hbar}\right)}, \quad (1)$$

where \vec{R}_n is the center position, λ is set to 19.8 nm and the whole wave function can be expanded on this basis,

$$\Psi(x,y,t) = \sum_{n=1}^N c_n(t) f_n(x,y). \quad (2)$$

The spacing between of the positions of the centers along the quantum wire is 20 nm and both quantum rings have same radius of 132 nm. The time evolution of the coefficients $C_n(t)$ can be solved from the time-dependent Schroedinger equation in matrix form,

$$i\hbar\dot{\mathbf{S}}(t) = \mathbf{H}\mathbf{c}(t), \tag{3}$$

where $\mathbf{S}_{kn} = \langle f_k | f_n \rangle$ and $\mathbf{H}_{kn} = \langle f_k | H | f_n \rangle$ are off-diagonal matrices which are very large to numerate by iterated method, nonetheless this calculation can be optimized by

$$i\hbar\dot{\mathbf{c}}(t) = \mathbf{M}\mathbf{c}(t), \tag{4}$$

where $\mathbf{M} = \mathbf{S}^{-1}\mathbf{H}$. We assume that $H = T + V_C$ is the Hamiltonian of the electron in the Gaussian confinement potential. Since \mathbf{S} and \mathbf{H} are Hermitian matrices, \mathbf{M} can be diagonalized by a unitary transformation,

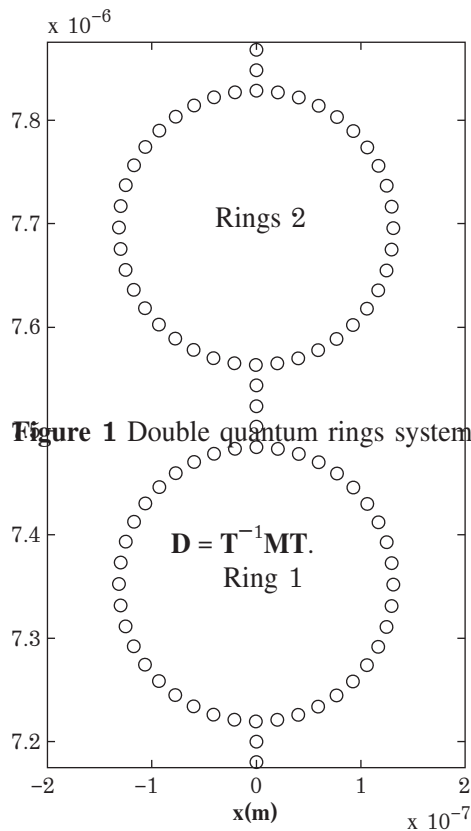


Figure 1 Double quantum rings system

(5)

Figure 1 Double quantum rings system

Here \mathbf{D} is diagonal and \mathbf{T} is the transformation matrix which can be found by solving eigenvalues and eigenvectors of \mathbf{M} . Equation becomes

$$i\hbar\mathbf{T}^{-1}\dot{\mathbf{c}}(t) = \mathbf{D}\mathbf{T}^{-1}\mathbf{c}(t). \quad (6)$$

Since \mathbf{T} is a time-independent matrix, we can rewrite in the form

$$\dot{\mathbf{C}}(t) = -\frac{i}{\hbar}\mathbf{D}\mathbf{C}(t). \quad (7)$$

The matrix equation (7) is a system of differential equations with the solution

$$\mathbf{C}(t) = \mathbf{C}(0)\exp(-i\mathbf{D}t/\hbar). \quad (8)$$

Now the probability amplitude $\mathbf{c}(t)$ is given by the transformation

$$\mathbf{c}(t) = \mathbf{T}\mathbf{C}(t). \quad (9)$$

The initial condition is the incident wave packet, which localized in the wire before injection to the first ring for 8th dots,

$$\Psi(x,y,0) = f_i(x,y)e^{iqy}, \quad (10)$$

where q is the wave number which is equal to 0.05 nm^{-1}

Results

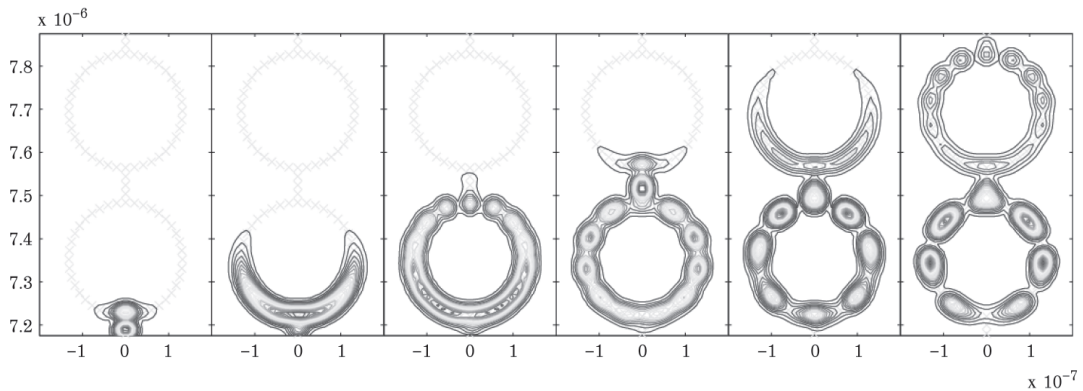


Figure 2 Probability for a Gaussian wave packet for zero magnetic field

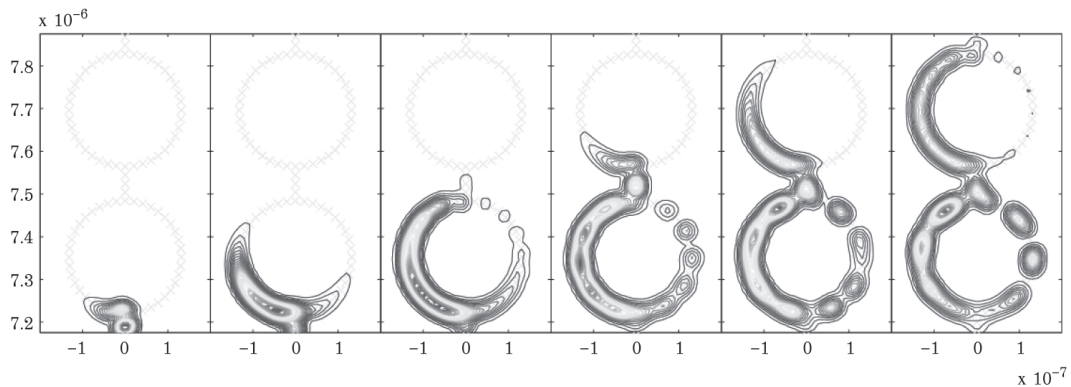


Figure 3 Probability for a Gaussian wave packet for magnetic field = 0.454 T

Figures 3-4 show the time evolution of wave packet at 2, 4, 6, 8, 10 and 12 picoseconds with the same magnetic field in both rings. The contour plots show probability, which are the square of coefficients of wave function. For the zero magnetic field [Figure 2] the wave packet being transferred through first ring and reduced in second ring. As the results, by applying the rings with large magnetic field in +z directions [Figure 3], the wave packet tends to the left arm of the rings by the Lorentz force. However the packet may be not pass the first ring [Figure 4] because of flux of magnetic field $\Phi = h/2e = 0.5\Phi_0$ causes destructive interference of the wave packet between the arms due to the Aharonov-Bohm effect. In this case, the effect of Lorentz force is small because of the low magnetic field.

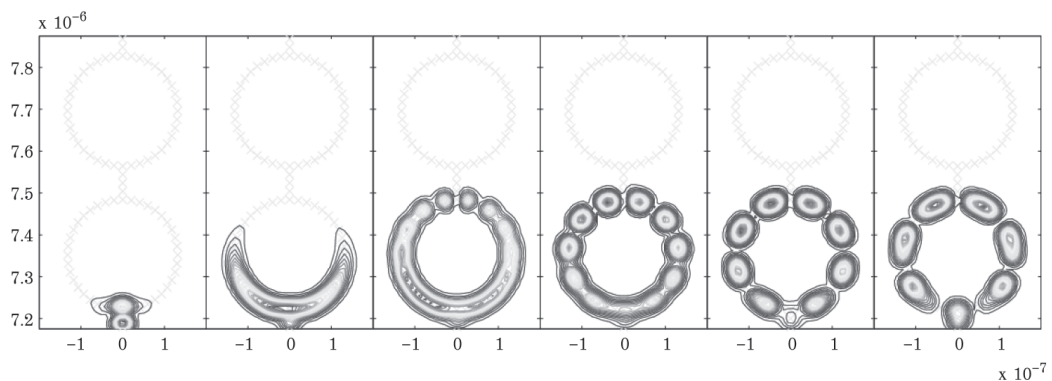


Figure 4 Probability for a Gaussian wave packet for magnetic field = 0.0378 T

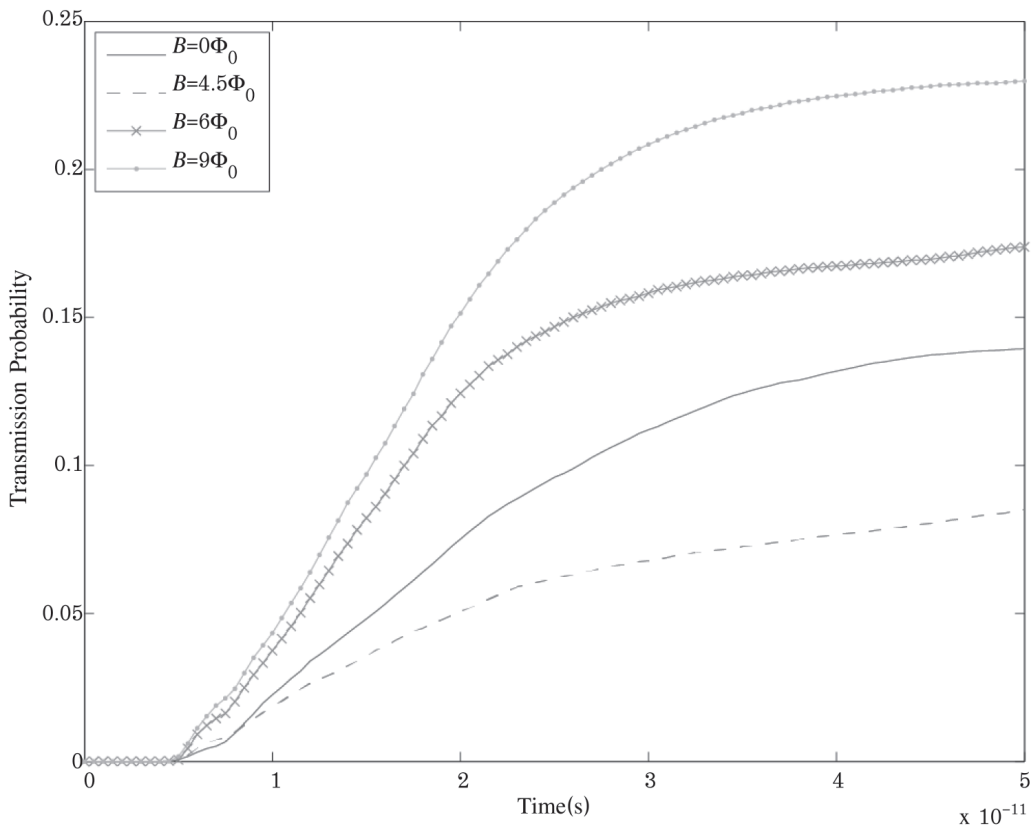


Figure 5 Transmission probability as a function of the time-evolution of the wave packet

The transmission probability defined by overall probability of the exit wire. In Figure 5, the graph for $B = 6\Phi_0$ represent the transmission probability of Figure 3. The graph for $B = 4.5\Phi_0$ shows slight inference effect (the complete one is at $B = 0.5\Phi_0$ as in Figure 4) which causes the transition to be lower than the case of zero magnetic field.

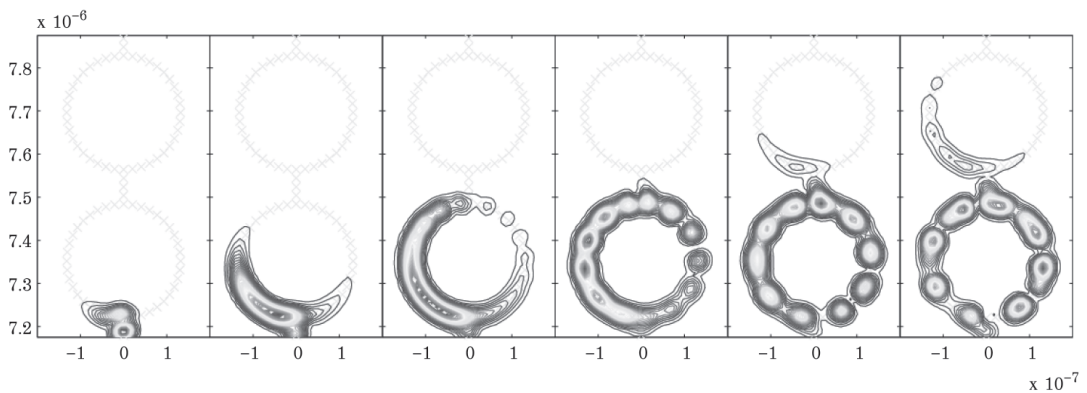


Figure 6 The case of magnetic field of first ring = 0.454 T and the second ring = 0.398 T

The case of different magnetic field between both rings is demonstrated by Figure 6. With $B_1 = 6\Phi_0$ and $B_2 = B_1 - 0.006$ the wave packet almost cannot be passed to the second ring and circulate back to the entrance. Thus we plot the transmission probability at time $t = 50$ ps by strict magnetic field of first ring, $B_1 = 0, 6\Phi_0, -6\Phi_0$ and vary magnetic field of the second ring. The result is shown in Figure 7. For zero magnetic field and magnetic field cases, the transmission probabilities have symmetry both side, because of the symmetry of Double quantum rings system. The overlap of the magnet lines seems to be increased from the lower line.

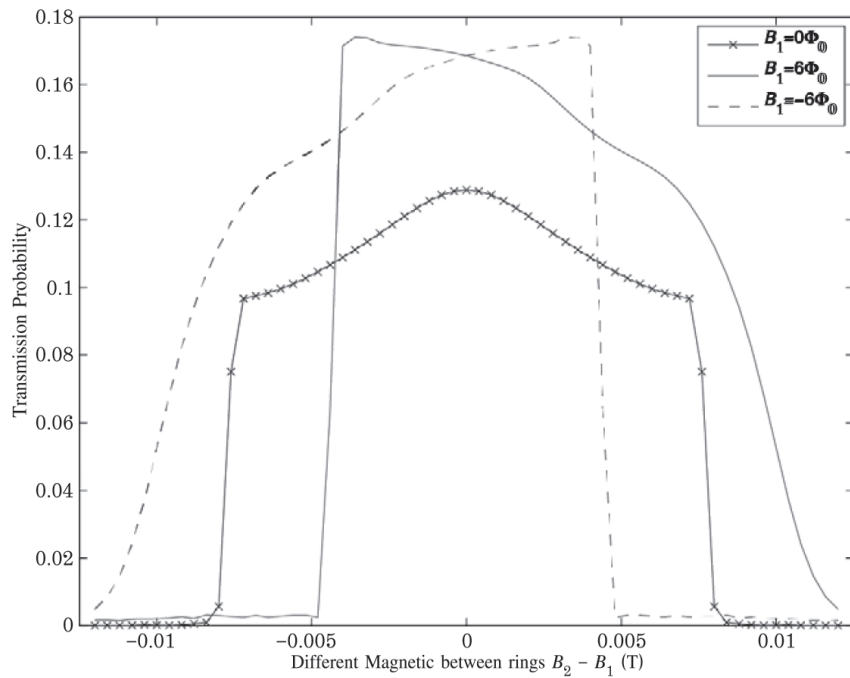


Figure 7 Transmission probability of the wave packet as a function of different magnetic fields between second and first rings with magnetic field of the first ring = 0, 0.454, -0.454

Conclusion and Discussion

We have solved the double quantum rings in static magnetic field with the system of differential equation by transformation to canonical form. The initial Gaussian wave packet can pass through the double quantum rings depending on conditions of applied magnetic field. The Aharonov-Bohm effect reduces probability of transmission to the exit wire because of the interference of phase shift between both arms. The different between magnetic field of both rings affects the transmission rate in such a way that, the difference is intensive enough, the wave packet cannot be passed the rings.

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