Science Essence Journal Vol. 42 No. 2 (2026) | 205-221

Research Article

Process of Mathematical Modeling: A Conceptual Framework for Mathematics Education

Worraphotchara Wongpratheep[1]*, Rungfa Janjaruporn[2] and Sermsri Thaithae[2]

Received: 10 March 2026
Revised: 5 May 2026
Accepted: 9 May 2026

ABSTRACT

Mathematical modeling is a crucial process in mathematics learning that connects mathematical knowledge with real-world situations. However, the modeling processes presented in the literature are diverse and fragmented, which has led to a lack of systematic understanding of the shared core of the mathematical modeling process. The purpose of this study is to synthesize the mathematical modeling process based on internationally recognized research and conceptual frameworks in mathematics education. A conceptual analysis and comparative examination of existing modeling frameworks were employed as the basis for the synthesis. The synthesis resulted in a mathematical modeling process consisting of five stages: understanding the real-world situation, simplifying, defining mathematical model, using mathematics, and explaining results. This process is conceptualized as dynamic and iterative, reflecting the nature of mathematical modeling as an epistemic activity in which learners continuously make decisions and revise their ideas. In addition, the synthesized framework helps organize knowledge related to the mathematical modeling process in a more coherent manner and elevates the role of simplification in making assumptions as a key theoretical component of modeling. This study contributes theoretically to mathematics education by offering a conceptual framework that can serve as a foundation for the design of instructional activities, the analysis of students' thinking processes, and future research on mathematical modeling.

Keywords: Mathematical modeling, Conceptual framework synthesis, Mathematics education

[1] Department of Subject Pedagogy, Faculty of Education, Ubon Ratchathani University, Ubon Ratchathani 34190, Thailand
[2] Department of Mathematics, Faculty of Science, Srinakharinwirot Universit, Bangkok 10110, Thailand
Corresponding author, email: Worraphotchara.w@gmail.com

Introduction

Mathematical modeling is widely recognized as a fundamental component of mathematics learning that connects mathematical knowledge with real-world situations and plays a crucial role in the development of students' analytical thinking, problem-solving, and decision-making abilities [1, 2]. Over the past several decades, research in mathematics education has proposed a variety of mathematical modeling frameworks, each emphasizing different stages and perspectives depending on learning contexts and researchers' theoretical orientations [3]. However, existing mathematical modeling frameworks remain fragmented and diverse in terms of the number of stages, the nature of the modeling process, and the emphasis placed on particular conceptual components, such as assumption making, the dynamic nature of modeling, and problem extension following the interpretation of solutions. This diversity may create challenges for teachers and researchers in selecting or applying an appropriate modeling framework that aligns with their instructional and research context [4, 5].

Recent scholarship suggests that mathematical modeling research has continued to expand in several important directions, including the conceptualization and fostering of modeling competencies, the use of digital technologies in modeling education, and international analyses of modeling goals and task characteristics [6-11]. These developments indicate that the field has become increasingly rich, but also more complex in terms of theoretical perspectives, instructional purposes, and learning environments. As a result, the need for a coherent synthesis of the modeling process remains significant in contemporary mathematics education.

Accordingly, the purpose of this study is to synthesize the mathematical modeling process based primarily on internationally recognized research and conceptual frameworks in mathematics education. The study aims to propose a systematic and coherent modeling framework that reflects the dynamic and iterative nature of mathematical modeling as an epistemic process. It is expected that the synthesized framework will function as both a theoretical and practical reference for instructional design, research, and the analysis of learners' modeling activity.

Research Questions of the Synthesis

The direction of the synthesis of the mathematical modelling process in this study was guided by a set of conceptually oriented research questions aimed at elucidating the core structure of mathematical modelling across diverse theoretical frameworks. These questions were not intended to yield empirical findings; rather, they functioned as an analytical lens through which theoretical knowledge was examined, organized, and integrated [12, 13].

The research questions underpinning this synthesis were as follows:

(1) Which stages or components of the mathematical modelling process recur with conceptual significance across established modelling frameworks proposed by scholars in mathematics education?

(2) Which process characteristics reflect the dynamic and iterative nature of mathematical modelling, particularly during transitions between the real-world domain and the mathematical domain?

(3) In what ways does the synthesized process framework demonstrate conceptual distinctiveness and added theoretical value when compared with existing modelling frameworks in the literature? [1, 3]

Research Focus and Scope of the Synthesis

The synthesis of the mathematical modeling process in this study adopts a process-oriented perspective, privileging the analysis and integration of modeling processes rather than focusing primarily on modeling competencies or assessment outcomes. Accordingly, the synthesis framework emphasizes the sequencing of cognitive actions, conceptually grounded decision-making, and transitional movements between the real-world domain and the mathematical domain during modeling activity [1, 3].

The scope of the synthesis encompasses research and theoretical frameworks on mathematical modeling situated within mathematics education, particularly at the primary, secondary, and tertiary levels. The review focuses primarily on internationally recognized scholarship that conceptualizes modeling as a pedagogical means for fostering mathematical understanding, analytical thinking, and meaningful connections between mathematics and reality.

This study does not seek to develop assessment criteria or measurement instruments; rather, it aims to systematize and clarify existing knowledge concerning the modeling process in order to establish a coherent conceptual framework to inform instructional design and future research [2, 5].

Theoretical Positioning of the Synthesized Framework

The synthesized framework of the mathematical modelling process proposed in this study is grounded in a constructivist perspective, which conceptualizes learning as the active construction of knowledge through learners' engagement with problematic situations, the formulation of assumptions, and the reflective interpretation of mathematical outcomes within real-world contexts. [4, 14] From this standpoint, mathematical modelling extends beyond the mere application of pre-existing formulas or procedures; rather, it constitutes an epistemic activity in which learners continuously exercise judgment, make selections, and engage in reasoned decision-making.

Furthermore, the synthesized framework aligns with perspectives that characterize mathematical modelling as a dynamic and iterative problem-solving process. Within such a process, learners may revisit, revise, or reorganize earlier stages when the constructed model proves insufficient to account for the real-world situation in a conceptually coherent manner [1, 3, 15]. The emphasis placed on assumption-making within the simplification phase also reflects the centrality of idealization, recognized as a foundational component of mathematical modelling in the theoretical accounts of Niss [5] and Kaiser and Sriraman [16].

Consequently, the synthesized process framework is not presented as a linear sequence of steps. Rather, it is articulated as a flexible conceptual structure that affords learners opportunities to exercise conceptual judgment and informed decision-making throughout the modelling activity. This orientation mirrors the complexity of real-world problem solving and resonates with contemporary theoretical perspectives in mathematics education.

Materials and Methods

The synthesis of the mathematical modeling process undertaken in this study was grounded in the principles of theoretical and integrative synthesis, which seek to consolidate knowledge from existing scholarship in order to generate a conceptually robust and practically meaningful framework [12-13]. In order to strengthen transparency and rigor, the synthesis process was guided by an explicit search and selection procedure adapted from Torraco's [6] guidelines for integrative literature reviews.

The literature search was conducted across major academic databases and scholarly search platforms relevant to mathematics education, including Scopus, ERIC, Web of Science, Google Scholar, and SpringerLink.

The search focused on publications addressing mathematical modeling processes, modeling cycles, conceptual frameworks, and related theoretical discussions in mathematics education.

The primary search terms included combinations of the following keywords: “mathematical modeling” OR “mathematical modelling”; “modeling process” OR “modelling process”; “modeling cycle” OR “modelling cycle”; “mathematics education”; “conceptual framework”; “problem posing”; and “assumption-making in mathematical modeling.”

The search considered publications from 1990 to 2025 in order to include both foundational works and recent scholarship. Particular attention was given to works published between 2020 and 2025 to ensure that the synthesis reflects current developments in the field, especially those related to contemporary learning environments [12-17].

The inclusion criteria were as follows: (1) publications that explicitly described stages, phases, or conceptual structures of the mathematical modeling process; (2) works situated within mathematics education; (3) studies or theoretical contributions that emphasized learners’ cognitive activity, decision-making, mathematization, interpretation, or validation; and (4) publications that were widely recognized, frequently cited, or theoretically influential in the field. The exclusion criteria included: (1) works focused primarily on assessment instruments without substantial discussion of modeling processes; (2) studies in engineering or applied sciences that did not address mathematics education; and (3) publications that mentioned modeling only in a peripheral or non-process-oriented manner.

Following the search process, the identified literature was screened for conceptual relevance and theoretical contribution. The selected works were then examined comparatively, with attention to process components that recurred across frameworks and to points of conceptual divergence. These recurring components were subsequently organized according to their functional roles within the modeling process: (1) understanding the real-world situation, (2) simplifying the situation through assumption-making, (3) defining the mathematical model, (4) using mathematics, and (5) explaining results.

Recent review studies have examined important subdomains of mathematical modeling research, such as modeling competencies, modeling goals, task characteristics, and assessment practices [6, 9, 11]. However, despite these advances, there remains a need for a focused conceptual synthesis that clarifies the shared structural core of the mathematical modeling process itself across diverse frameworks. The present study addresses this need by concentrating specifically on process structure and conceptual integration.

This analytical approach aligns with the propositions of Blum and Leiß [15] and Stillman et al. [3], who advocate identifying common structural cores across diverse modeling frameworks. In addition, the synthesis considered not only the presence of process stages, but also the ways in which particular frameworks emphasized recursion, validation, simplification, and post-solution reformulation. This allowed the present study to move beyond simple aggregation toward a theoretically differentiated synthesis.

To enhance conceptual clarity, the synthesis also foregrounded the dynamic nature of mathematical modeling. Particular consideration was given to iterative movements and recursive transitions among stages, as well as to the central roles of assumption formulation and model validation in ensuring the reasonableness of solutions. These features reflect the inherently cyclical and reflective character of mathematical modeling as enacted in authentic learning contexts [5, 14].

Criteria for Literature Selection

The selection of documents for this synthesis was guided by the principle of conceptual relevance together with the need for transparent and theoretically grounded inclusion. Rather than attempting exhaustive coverage of all publications mentioning mathematical modeling, the study prioritized works that have made substantive contributions to conceptualizing the modeling process in mathematics education. The corpus therefore comprised widely cited research articles, scholarly books, book chapters, and theoretical contributions within the field [2, 17].

The criteria for document selection included the following: (1) Publications that explicitly articulate a process framework of mathematical modeling; (2) Works that foreground learners' cognitive processes, decision-making, mathematization, interpretation, or validation during modeling activity; (3) Contributions that are internationally recognized, theoretically influential, or foundational within mathematics education scholarship; and (4) Recent publications that help situate the synthesized framework in relation to current developments in the field, including technology-rich and emerging AI-supported learning environments [12-17].

Although the synthesis was conceptually oriented, the literature selection process was designed to enhance rigor by making explicit the search scope, inclusion logic, and analytical basis for selecting source texts. This approach is consistent with integrative review methodology, in which conceptual relevance is combined with transparent scholarly procedures [12].

Results and Discussion

Definition

Mathematical model refers to a structured system of variables, relationships, assumptions, and mathematical representations that is constructed to describe, explain, or predict aspects of a real-world phenomenon or situation. A mathematical model does not consist merely of isolated symbols or representations; rather, it organizes mathematical elements into a coherent form that captures essential features of the phenomenon under investigation.

For example, a linear cost model relating total production cost to the number of units produced, a population growth model describing change over time, or a motion model expressing distance as a function of time may each be considered mathematical models. In contrast, equations, functions, graphs, and tables are more appropriately understood as forms of representation through which a model may be expressed, explored, or communicated.

Mathematical modeling refers to a dynamic process of investigating real-world situations through the construction and use of mathematical representations. It involves understanding and interpreting a real-world context, transforming that context into mathematical form, and systematically refining conditions to make the problem analytically tractable. The process requires selecting relevant mathematical knowledge, identifying and defining variables, constructing a mathematical model, and generating solutions through mathematical operations. The resulting solutions are then interpreted and translated back into the real-world context, evaluated for their plausibility and contextual appropriateness, and, where relevant, extended to related situations through reformulation or problem posing.

In this study, the mathematical modeling process is conceptualized as comprising five interrelated stages:

1) Understanding a real-world situation–comprehending and clarifying the features and constraints of the given context.

2) Simplifying– formulating assumptions and reducing complexity to render the situation mathematically manageable.

3) Defining a mathematical model–identifying variables, establishing relationships, and constructing a formal mathematical representation.

4) Using mathematics–performing mathematical procedures to analyze the model and derive solutions.

5) Explaining results–interpreting and validating solutions in relation to the original real-world situation, while also considering possible reformulation, extension, or problem posing based on the constructed model.

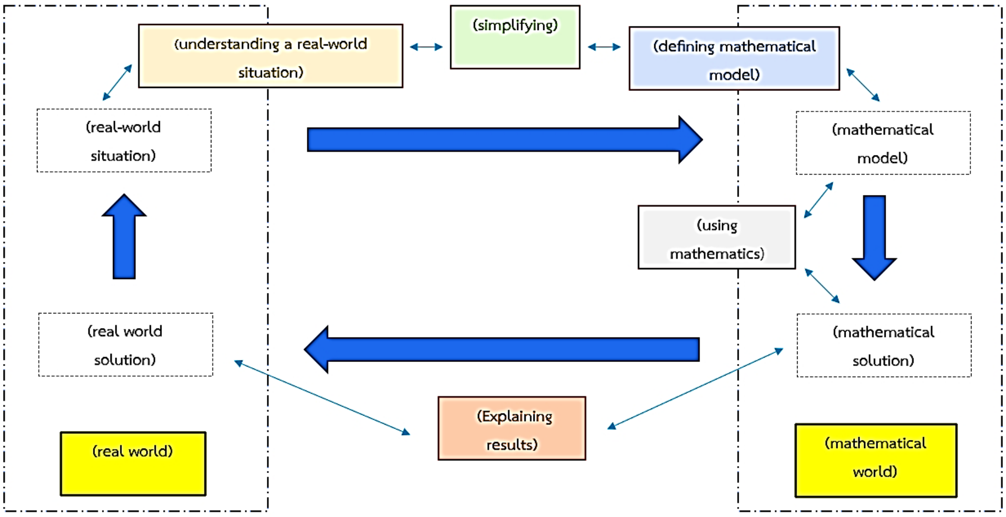

Figure 1 The synthesized framework of the mathematical modeling process developed in this study.

Stage 1: Understanding a Real-World Situation

The first stage involves comprehensively understanding the real-world situation. At this stage, learners engage in an investigative inquiry process to gather additional information, seek explanations, and identify underlying relationships within the situation. This inquiry-oriented engagement supports the exploration of relevant data, clarification of contextual demands, and careful consideration of the conditions and constraints embedded in the real-world context. Learners may employ representational tools—such as constructing tables—to organize essential information and examine potential advantages and limitations inherent in the situation. Through this systematic examination, critical elements of the context become more explicit, thereby laying the foundation for subsequent identification and definition of variables in later stages of the modelling process.

Stage 2: Simplifying

The second stage involves simplifying the real-world situation by critically examining the factors and information that do not substantially contribute to solving the problem and excluding such elements from the set of defined variables. Learners then reconsider the remaining factors within the situation, focusing on those that are essential for constructing a mathematically meaningful representation. This stage is not merely a process of reducing information; rather, it involves making theoretically and contextually justified assumptions that shape the structure, scope, and interpretive power of the emerging model. This stage also entails identifying relevant mathematical knowledge that may support the resolution of the problem. Through this selective reduction and refinement of contextual elements, the situation becomes more analytically manageable, enabling learners to move toward formal mathematical structuring in subsequent stages.

Stage 3: Defining a Mathematical Model

The third stage involves defining a mathematical model by transforming the relevant conditions and essential factors identified in the previous stage into variables or unknown quantities. Through this formalization process, key aspects of the situation are represented in mathematical terms, enabling the construction of a mathematical model that can be used to derive solutions to the real-world problem. At this stage, a critical transition occurs from the real-world domain to the mathematical domain. This movement represents the process of mathematization, in which contextual elements are systematically structured into formal mathematical relationships.

Stage 4: Using Mathematics

The fourth stage involves applying the constructed mathematical model to derive solutions to the real-world situation. At this stage, learners operate within the mathematical domain, performing appropriate mathematical procedures and analytical operations based on the established relationships among variables. Through these mathematical processes, results are obtained and conclusions are formulated within the mathematical framework. The outcomes generated at this stage represent solutions situated in the mathematical domain, which will subsequently require interpretation in relation to the original real-world context.

Stage 5: Explaining Results

The fifth stage involves interpreting the mathematical results in relation to the real-world situation. At this stage, solutions obtained within the mathematical domain are critically examined in terms of their plausibility, feasibility, and contextual appropriateness before being translated back into the original real-world context. This interpretive activity also includes evaluating whether the assumptions underlying the model remain reasonable in light of the obtained results. Once a meaningful real-world interpretation is established, learners may engage in mathematical problem posing by reformulating conditions, generating analogous situations, or extending the original problem to new contexts. In this sense, problem posing is not treated as an external add-on, but as a theoretically meaningful continuation of modeling activity through which the explanatory reach of a model may be tested, adapted, or expanded [18-20]. The previously constructed mathematical model is then reconsidered, and, where necessary, revised or refined to accommodate newly formulated problems. Through this process, the model may be adapted or extended to support further analysis and solution generation.

Within the simplification stage, assumption-making plays a pivotal role in reducing the complexity of the real-world situation. Assumptions function as a conceptual framework for selecting relevant factors, delineating the boundaries of the problem, and establishing relationships among variables. Accordingly, simplification should not be understood as the mere removal of unnecessary details, but as an epistemic act through which the modeler constructs an idealized yet workable version of reality. This perspective aligns with the arguments of Niss and Kaiser [2, 5] and Sriraman [16], who emphasize that simplification in mathematical modeling entails intentional abstraction through the formulation of assumptions rather than arbitrary data reduction. The assumptions introduced at this stage directly influence the plausibility and validity of the resulting mathematical model. Consequently, these assumptions must be critically examined and, where necessary, refined during the stages of interpretation and explanation of results [14].

Although the synthesized mathematical modeling process is presented in a sequenced format for conceptual clarity, it should not be construed as a linear progression in practice. Rather, modelling unfolds as a dynamic and iterative process in which the modeler may continuously revisit, revise, or refine earlier stages. Such recursive movement becomes particularly necessary when mathematical results fail to provide a plausible or contextually coherent explanation of the real-world situation. This iterative character is consistent with the modelling cycle articulated by Blum and Leiß [15], as well as with the perspectives of Stillman, Brown, and Galbraith [3], who conceptualize mathematical modelling as a recursive process requiring ongoing validation, decision-making, and adaptive refinement. In this sense, the modelling process entails sustained epistemic regulation, where learners monitor the adequacy of their models and make informed adjustments in response to emerging inconsistencies or limitations.

The synthesized framework of the mathematical modeling process in this study is grounded in an epistemological stance that views mathematical knowledge not as fixed or absolute truth, but as knowledge constructed through processes of explanation, assumption-making, and the examination of the plausibility of models within specific contexts [5, 14]. From this perspective, a mathematical model is not required to represent the "most correct" version of reality; rather, it is understood as a contextually appropriate and functionally adequate representation under the assumptions operative at a given moment. The emphasis placed on simplification and assumption-making within the modelling process reflects the principle of idealization, widely recognized as a central feature of mathematical modelling [5, 16]. Assumptions function as epistemic tools that enable the modeler to manage the inherent complexity of real-world situations. At the same time, these assumptions remain open to scrutiny, revision, and refinement through iterative cycles of interpretation and validation, particularly when they fail to account plausibly for the situation under consideration.

Table 1 Summary of the analysis and synthesis of the mathematical modeling process.

educator	**synthesized process stages**				
	understanding a real-world situation	**simplifying**	**defining mathematical model**	**using mathematics**	**explaining results**
Swetz & Hartzler, 1991 [21]	engage with authentic real-world situations		establish relationships among variables	analyze the data and relevant variables	derive mathematically appropriate conclusions
Giordano et al., 2014 [22]	simplification		analysis		interpretation and validation
Carreira, Amado, & Lecoq, 2011 [23]	consider empirically observable reality	define the scope of inquiry	construct a mathematical model	construct a mathematical representation of the solution	
Greefrath, 2011 [24]	transform the real-world situation into a situational model in order to establish a mathematically operative model		translate the situation into the mathematical domain	utilize technological tools to obtain computational results	interpret mathematical results within the real-world context

Table 1 Summary of the analysis and synthesis of the mathematical modeling process. *(cont.)*

educator	**synthesized process stages**				
	understanding a real-world situation	**simplifying**	**defining mathematical model**	**using mathematics**	**explaining results**
Galbraith, 2013 [14]	develop an informed understanding of the real-world problem situation	formulate mathematically appropriate investigative questions grounded in the contextual setting	construct a mathematical model based on simplifying assumptions, drawing upon relevant mathematical knowledge and concepts	analyze the model and compare the resulting mathematical conclusions with empirical or contextual realities	revise and refine the model if it proves insufficient for addressing the problem adequately
Ho, 2015 [25]	examine the real-world situation	formulate a corresponding mathematical problem	reason toward mathematically grounded conclusions that address the formulated problem	interpret the results within the original context	
Suh & Seshaiyer, 2017 [26]	explore possible ways in which the problem may be resolved	formulate simplifying assumptions to render the situation more conceptually accessible	construct a mathematical model and derive its solution	analyze and interpret the obtained results	
Borromeo Ferri, 2018 [27]	read and develop a thorough understanding of the problem situation	simplify the situation by reducing contextual complexity	conceptualize the situation in mathematical terms	mathematize the situation through formal representation	interpret the results, evaluate their adequacy, and communicate the findings
Blum & Leiß, 2007 [15]	construction of a situational model	simplification through explicit assumptions	mathematization of the situation	engagement in formal mathematical work	interpretation of results, evaluation of contextual adequacy, and application of the solution to explain the real-world situation

Table 1 Summary of the analysis and synthesis of the mathematical modeling process. *(cont.)*

educator	**synthesized process stages**				
	understanding a real-world situation	**simplifying**	**defining mathematical model**	**using mathematics**	**explaining results**
Dindyal, 2010 [28]	employ mathematical reasoning to develop an understanding of the problem situation	formulate simplifying assumptions to reduce contextual complexity		derive a mathematically formulated solution	interpret the obtained solution, evaluate its contextual adequacy, and apply the conclusions to the real-world situation
Niss, 2013 [5]	examine the situation within a non-mathematical or extra-mathematical context	identify salient and operationalizable features of the situation and formulate questions grounded in an idealized representation	engage in mathematical thinking and transform the situation into the mathematical domain (mathematization)	derive solutions to mathematically relevant and interconnected questions.	interpret and recontextualize the mathematical reasoning to generate conclusions applicable to the real-world situation
Stillman, Brown, & Galbraith, 2013 [3]	comprehension and structuring		model construction	mathematical work	interpretation, evaluation, and communication
Arseven, 2015 [29]	develop an understanding of the real-world situation and reconstruct it as a structured situational model		formulate the situation as a real-world problem, engage in mathematical thinking, and transform it into a formal mathematical model	mobilize relevant mathematical knowledge to derive conclusions within the mathematical domain	interpret and translate the mathematical conclusions back and forth between the mathematical domain and the real-world context
Teague, Godbold, Malkevitch, & Kooij, 2016 [30]	identify and define the problem that requires resolution	-	formulate explicit assumptions and determine the key variables	mathematize the situation in order to derive a solution	analyze and evaluate the model and its solutions, refine and improve the model as necessary, and apply the mathematical model in practice

Table 1 Summary of the analysis and synthesis of the mathematical modeling process. *(cont.)*

educator	synthesized process stages				
	understanding a real-world situation	**simplifying**	**defining mathematical model**	**using mathematics**	**explaining results**
Zbiek, 2016 [31]	develop a thorough understanding of the problem	-	formulate an appropriate mathematical expression or formula for computation		interpret the results, evaluate their reasonableness, and communicate the findings
Zeytun, Cetinkaya, & Erbas, 2017 [32]	examine the real-world problem situation		formulate the problem mathematically by constructing an appropriate expression or formula	perform symbolic manipulation or algebraic transformation	interpret the results and evaluate their reasonableness
Stender, Krosanke, & Kaiser, 2017 [33]	develop an understanding of the real-world situation	reduce contextual complexity through deliberate simplification	reconstruct the situation as a structured situational model and engage in mathematical reasoning to transform it into a formal mathematical model	derive solutions to the model by mobilizing relevant knowledge and applying formal mathematical processes	interpret the results in relation to the real-world context and evaluate their correctness and contextual plausibility
Bora & Ahmed, 2019 [34]	interpret the problem situation	-	employ abstract knowledge to transform the problem representation into a mathematical model or formal representation	apply mathematical processes to draw inferences	interpret the results of the mathematical model in order to generate conclusions about the real-world situation
Viseu, Martins, & Leite, 2020 [35]	develop an understanding of the situation under investigation		construct a mathematical model	apply mathematical processes to derive results	interpret the results, evaluate the model, and connect the conclusions to the problem situation

Table 1 shows that, despite differences in terminology and stage structure, the reviewed frameworks share a common conceptual core. Across the literature, mathematical modeling consistently involves: (1) engaging with and understanding a real-world situation, (2) reducing or structuring complexity through simplification or assumption-making, (3) constructing or defining a mathematical model, (4) carrying out mathematical work to obtain results, and (5) interpreting, evaluating, or communicating those results in relation to the original context. At the same time, the frameworks differ in the degree to which they explicitly emphasize simplification, recursive movement, validation, communication, and post-solution refinement. These recurring similarities and meaningful differences provided the analytical basis for synthesizing the five-stage framework proposed in this study.

Conceptual Distinctiveness and Contributions of the Synthesized Process

When compared with previously established frameworks of the mathematical modeling process, the framework synthesized in this study demonstrates several notable points of distinction. However, these distinctions should be understood not as a rejection of prior frameworks, but as an attempt to consolidate their shared conceptual core while making certain underemphasized features more explicit.

First, compared with Blum and Leiß's modeling cycle, the synthesized framework more explicitly foregrounds simplification as an epistemic stage centered on assumption-making rather than treating it only as a transitional feature within movement toward mathematization. Second, compared with the framework of Stillman, Brown, and Galbraith, the present synthesis articulates a more sharply differentiated five-stage structure that clarifies the functional role of each stage while preserving the recursive nature of movement across stages. Third, the synthesized framework extends beyond solution interpretation by explicitly incorporating mathematical problem posing and problem extension as theoretically meaningful outcomes of modeling activity.

These distinctions suggest that the present framework contributes not merely by reducing several models into a simplified sequence, but by offering a conceptually clarified synthesis that highlights (1) the epistemic significance of assumption-making, (2) the structural importance of iteration and revision, and (3) the generative potential of modeling through post-solution reformulation. In this sense, the framework advances prior literature by making visible dimensions that are often implicit, dispersed, or only briefly mentioned in existing models.

These refinements align with the principles articulated by Torraco [12] and Corbin and Strauss [36], who argue that theoretical work must demonstrate clear conceptual value and differentiation from prior frameworks. Within the context of mathematical modeling, this orientation also resonates with the perspective of Blum and Stillman [4], who emphasize that modeling frameworks should reflect the complexity of real-world problem solving while systematically accounting for learners' cognitive processes.

Table 2 Analytical comparison between major modeling frameworks and the synthesized framework.

Dimension	Blum & Leiß	Stillman et al.	Synthesized framework in this study
Overall orientation	Modeling cycle connecting reality and mathematics	Process-oriented framework for teaching and learning modeling	Integrative conceptual synthesis of recurrent modeling processes
Stage structure	Multi-step modeling cycle	Broad process structure with recursive movement	Five clearly differentiated stages
Role of simplification	Present, especially through assumptions	Present but less explicitly isolated as a separate epistemic stage	Explicitly foregrounded as a distinct stage centered on assumption-making
Movement across stages	Cyclical and iterative	Recursive and adaptive	Dynamic, iterative, and recursively revisable

Table 2 Analytical comparison between major modeling frameworks and the synthesized framework. *(cont.)*

Dimension	Blum & Leiß	Stillman et al.	Synthesized framework in this study
Emphasis on interpretation/evaluation	Strong	Strong	Strong, with added attention to validation of assumptions
Problem posing/extension	Limited or implicit	Limited or implicit	Explicitly included in the explaining results stage
Main contribution	Foundational modeling cycle	Pedagogically significant process account	Conceptual clarification and synthesis across frameworks

As shown in Table 2, the synthesized framework retains the major strengths of established modeling frameworks while sharpening several theoretical dimensions that are particularly important for conceptual clarity. Most notably, it makes simplification more visible as an epistemic act, treats recursive revision as a central structural feature, and recognizes problem posing as a legitimate continuation of modeling activity after interpretation and validation.

The importance of making the modeling process conceptually explicit is further strengthened by recent research on digitally mediated modeling activity. Contemporary studies indicate that learners and pre-service teachers often use digital technologies at multiple stages of the modeling process, but may also struggle with critical evaluation, validation, and the meaningful transfer of mathematical knowledge when technology use becomes overly procedural [7-10]. This suggests that a clear process framework remains especially important in current modeling environments shaped by digital tools.

Limitations

Although the synthesized mathematical modeling process demonstrates conceptual coherence and alignment with established modelling frameworks, several limitations should be acknowledged.

First, the proposed framework is primarily conceptual in nature, having been developed through theoretical synthesis of prior research. While preliminary empirical examination has been conducted through limited implementation in instructional contexts, the framework has not yet undergone extensive validation across diverse classroom settings or large-scale field studies. Further empirical investigation is therefore required to examine its robustness, applicability, and instructional impact.

Second, the synthesized process framework is situated predominantly within the domain of mathematics education. As such, it may not fully capture the distinctive characteristics of modelling practices in other disciplines, such as engineering or applied sciences, where contextual constraints—including time limitations, technical specifications, and domain-specific tools—may shape the modelling process differently. [4] Consequently, the application of this framework in alternative disciplinary contexts should involve careful adaptation in accordance with learners' characteristics and instructional objectives.

Implications for Research and Instruction

The synthesized mathematical modeling process may serve as a conceptual framework for designing learning activities aimed at developing students' mathematical modelling competence. In instructional contexts, the framework can guide the structuring of modelling tasks to ensure that learners engage in each critical stage of the modelling process, from contextual inquiry to model refinement.

From a research perspective, the framework may also function as an analytical lens for examining learners' cognitive processes and decision-making during modelling activity. This orientation is consistent with the principles of design-based research, which emphasize the iterative interplay between theoretical constructs and classroom practice [37].

Moreover, the synthesized process may inform the development of evaluative criteria for assessing the quality of mathematical models, particularly in terms of plausibility, contextual coherence, and the appropriateness of underlying assumptions. Such applications align with the proposals of Kaiser et al. [2] and Blum [4], who advocate the use of modelling frameworks not only for instructional design, but also for research and assessment purposes.

This implication is particularly relevant in light of recent work on technology-supported modeling environments and teacher education. Emerging evidence suggests that well-designed technology-supported learning environments can strengthen modeling competencies, while research on digital modeling processes also highlights the need for explicit instructional guidance to support validation, interpretation, and critical engagement with technological outputs [7-10]. In addition, recent systematic review work on the measurement of mathematical modeling indicates that clearer process frameworks can support the development of more coherent evaluative criteria and assessment designs [9].

Conclusions

This study presents a theoretical synthesis of the mathematical modeling process through the integration of established modeling frameworks from mathematics education literature. The synthesis results in a clearly articulated five-stage process framework that reflects the dynamic nature of mathematical modeling as an activity grounded in assumption-making, decision-making, and ongoing validation.

The study contributes theoretically by systematizing fragmented knowledge concerning the modeling process and by clarifying several dimensions that are often treated only implicitly in prior frameworks, particularly the epistemic role of simplification, the recursive character of modeling, and the place of problem posing as an extension of explanatory work.

Beyond its theoretical value, the framework demonstrates potential applicability as a guiding structure for instructional design, research analysis, and the assessment of mathematical modeling competence. Future empirical research that implements this framework across diverse instructional settings will be essential to further examine its contextual adaptability, practical feasibility, and explanatory power in authentic learning environments.

The relevance of such a framework is further underscored by recent scholarship showing that mathematical modeling is increasingly shaped by diverse goals, digital technologies, and evolving instructional environments [6-11]. In this context, a coherent synthesis of the modeling process may provide useful conceptual support for both contemporary research and classroom practice.

Directions for Future Research

Although the synthesized mathematical modeling process demonstrates theoretical and practical potential, several avenues for future research emerge from the present study.

First, broader empirical investigations are needed to implement the synthesized framework as a basis for designing modelling-oriented learning activities and to examine how learners enact each stage of the process in authentic classroom settings. Large-scale and cross-contextual studies would further illuminate the robustness and instructional viability of the framework across diverse educational environments.

Second, the framework may serve as an analytic lens for examining learners' cognitive processes, decision-making, and assumption-making during mathematical modelling. Such applications could contribute to the development of qualitative analytical tools or coding schemes that are aligned with the epistemic and iterative nature of modelling [2, 3].

Additionally, future research may extend the application of this framework to varied learning contexts, including different educational levels and interdisciplinary settings. Investigating its adaptability and limitations across broader domains may provide deeper insight into the contextual sensitivity and generalizability of the modelling process framework.

Theoretical Contributions of the Study

This study offers several significant theoretical contributions. First, it consolidates and systematizes fragmented knowledge concerning the mathematical modeling process within the existing literature. By synthesizing recurring structural elements across established frameworks [1-3], the study articulates a coherent five-stage process that captures their shared conceptual core. The formulation of this structured framework reduces conceptual ambiguity and enhances clarity in both theoretical interpretation and pedagogical application.

Second, the study reconceptualizes the role of simplification and assumption-making. Rather than treating them as minor procedural steps, it positions these elements as central epistemic components in the construction and validation of mathematical models. This reframing underscores the theoretical significance of idealization and assumption-based reasoning in shaping the structure, scope, and explanatory adequacy of models.

Furthermore, by foregrounding the dynamic and iterative character of modelling, the study advances the understanding of mathematical modelling as a process-oriented and decision-intensive activity rather than a linear procedural sequence. This orientation aligns with contemporary perspectives in mathematics education that conceptualize modelling as situated, reflective, and responsive to the complexities of real-world problem solving [2-4].

References

1. Brown JP. Mathematical modelling in teacher education—overview. In: Kaiser G, Blum W, Borromeo Ferri R, Stillman G, editors. Trends in teaching and learning of mathematical modelling: ICTMA14. Dordrecht: Springer; 2011. p. 443-446.
2. Kaiser G, Blomhøj M, Sriraman B. Towards a didactical theory for mathematical modelling. ZDM Math Educ. 2011;43(1):1-14.
3. Stillman GA, Brown JP, Galbraith P. Research into the teaching and learning of mathematical modelling. In: Stillman GA, Blum W, Kaiser G, editors. Teaching mathematical modelling: connecting to research and practice. Dordrecht: Springer; 2013. p. 1-12.

4. Blum W, Stillman G. Teaching mathematical modelling: connecting to research and practice. In: Stillman GA, Blum W, Biembengut MS, editors. Mathematical modelling in education research and practice. Cham: Springer; 2015. p. 3-16.
5. Niss M. Models and modelling in mathematics education. In: Stillman GA, Blum W, Kaiser G, editors. Teaching mathematical modelling: Connecting to research and practice. Dordrecht: Springer; 2013. p. 1-21.
6. Cevikbas M, Kaiser G, Schukajlow S. A systematic literature review of the current discussion on mathematical modelling competencies: state-of-the-art developments in conceptualizing, measuring, and fostering. Educ Stud Math. 2022;109(2):205-36.
7. Cevikbas M, Greefrath G, Siller HS. Advantages and challenges of using digital technologies in mathematical modelling education: a descriptive systematic literature review. Front Educ. 2023;8:1142556.
8. Drijvers P, Sinclair N. The role of digital technologies in mathematics education: purposes and perspectives. ZDM Math Educ. 2024;56(2):239-48.
9. Hidayat R, Adnan M, Abdullah MFNL, Safrudiannur. A systematic literature review of measurement of mathematical modeling in mathematics education context. EURASIA J of Math Sci Tech Ed. 2022;18(5):em2108.
10. Kandemir MA, Eryilmaz N. Pre-service mathematics teachers' modelling processes within model eliciting activity through digital technologies. Acta Psychol. 2025;256:104990.
11. Krawitz J, Schukajlow S, Yang X, Geiger V. A systematic review of international perspectives on mathematical modelling: modelling goals and task characteristics. ZDM Math Educ. 2025;57(2-3):193-212.
12. Torraco RJ. Writing integrative literature reviews: guidelines and examples. Hum Resour Dev Rev. 2005;4(3):356-67.
13. Walker LO, Avant KC. Strategies for theory construction in nursing. 6th ed. Boston: Pearson; 2019.
14. Galbraith P. Mathematical modelling: context, complexity and competence. In: Stillman GA, Blum W, Kaiser G, editors. Teaching mathematical modelling: connecting to research and practice. Dordrecht: Springer; 2013. p. 17-34.
15. Blum W, Leiß D. How do students and teachers deal with modelling problems? In: Haines C, Galbraith P, Blum W, Khan S, editors. Mathematical modelling (ICTMA 12): education, engineering and economics. Chichester: Horwood; 2007. p. 222-31.
16. Kaiser G, Sriraman B. A global survey of international perspectives on modelling in mathematics education. ZDM - Int J Math Educ. 2006;38(3):302-10.
17. Blum W. Quality teaching of mathematical modelling: what do we know, what can we do? In: Cho SJ, editor. The proceedings of the 12th international congress on mathematical education. Cham: Springer; 2015. p. 73-96.
18. Silver EA. On mathematical problem posing. For Learn Math. 1994;14(1):19-28.
19. Cai J, Hwang S. Generalized and generative thinking in U.S. and Chinese students' mathematical problem solving and problem posing. J Math Behav. 2002;21(4):401-21.
20. Blum W, Niss M. Origin and development of the notion of mathematical modelling competency/competencies. In: International perspectives on the teaching and learning of mathematical modelling. Cham: Springer; 2024. p. 185-200.

21. Swetz F, Hartzler JS. Mathematical modeling in the secondary school curriculum: a resource guide of classroom exercises. Reston: National Council of Teachers of Mathematics; 1991.
22. Giordano FR, Fox WP, Horton SB. A first course in mathematical modeling. 5th ed. Boston: Cengage; 2014.
23. 2Carreira S, Amado N, Lecoq F. Mathematical modelling of daily life in adult education: focusing on the notion of knowledge. In: Kaiser G, Blum W, Borromeo Ferri R, Stillman G, editors. Trends in teaching and learning of mathematical modelling. Dordrecht: Springer; 2011. p. 199-210.
24. Greefrath G. Using technologies: New possibilities of teaching and learning modelling-overview. In: Kaiser G, Blum W, Borromeo Ferri R, Stillman G, editors. Trends in teaching and learning of mathematical modelling: ICTMA14. Dordrecht: Springer; 2011. p. 301-4.
25. Ho SY. Learning through designing a tent. In: Niss M, Lee PY, Kilpatrick J, editors. Mathematical modelling: from theory to practice. Singapore: World Scientific; 2015. p. 205-20.
26. Suh JM, Seshaiyer P. Modeling mathematical ideas: developing strategic competence in elementary and middle school. Lanham: Rowman & Littlefield; 2017.
27. Borromeo Ferri R. Learning how to teach mathematical modeling in school and teacher education. Cham: Springer; 2018.
28. Dindyal J. Word problems and modelling in primary school mathematics. In: Kaur B, Dindyal J, editors. Mathematical applications and modelling: Yearbook 2010. Singapore: World Scientific; 2010. p. 94-111.
29. Arseven A. Mathematical modelling approach in mathematics education. Univers J Educ Res. 2015;3(12):973-80.
30. Teague D, Godbold L, Malkevitch J, van der Kooij H. Mathematical modeling in high school: grades 9 through 12. In: Garfunkel S, Montgomery M, editors. GAIMME: Guidelines for assessment and instruction in mathematical modeling education. Bedford: COMAP & SIAM; 2016. p. 45-70.
31. Zbiek RM. Supporting teachers' development as modelers and teachers of modelers. In: Hirsch CR, McDuffie AR, editors. Annual perspectives in mathematics education 2016: mathematical modeling and modeling mathematics. Reston: National Council of Teachers of Mathematics; 2016. p. 263-72.
32. Zeytun AS, Cetinkaya B, Erbas AK. Understanding prospective teachers' mathematical modeling processes in the context of a mathematical modeling course. EURASIA J Math Sci Tech Ed. 2017;13(3):691-722.
33. Stender P, Krosanke N, Kaiser G. Scaffolding complex modelling processes: an in-depth study. In: Stillman GA, Blum W, Kaiser G, editors. Mathematical modelling and applications: crossing and researching boundaries in mathematics education. Cham: Springer; 2017. p. 467-77.
34. Bora A, Ahmed S. Mathematical modeling: an important tool for mathematics teaching. Int J Res and Anal Rev. 2019;6(2):252-6.
35. Viseu F, Martins PM, Leite L. Prospective primary school teachers' activities when dealing with mathematics modelling tasks. J Math Educ. 2020;11(2):301-18.
36. Corbin J, Strauss A. Basics of qualitative research: techniques and procedures for developing grounded theory. 4th ed. Thousand Oaks: SAGE; 2015.
37. Cobb P, Confrey J, diSessa A, Lehrer R, Schauble L. Design experiments in educational research. Educ Res. 2003;32(1):9-13.