Review Article

The Nature and Significance of Mathematics from Contemporary Viewpoints

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ABSTRACT

This paper explores the nature and significance of mathematics, presenting it as a systematic and logical study of patterns in nature. Mathematics can be conceptualized as a pyramid consisting of three layers. The first layer is pure mathematics, which focuses on the study of abstract objects and concepts. The second layer is applied mathematics, dedicated to the development and application of mathematical methods to address specific problems. The final layer involves the applications of mathematics, where established results from pure or applied mathematics are utilized to solve concrete, real-world problems. A deep appreciation of the importance and beauty of abstract patterns requires engaging in research within pure or applied mathematics. To undertake such research, the fundamentals of pure mathematics are essential. Advances in mathematical research often lead to the development of new theories or innovative techniques for problem-solving.

Keywords: Pure mathematics, Applied mathematics, Applications of mathematics, Mathematical research.

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Introduction

In recent years, universities worldwide have increasingly prioritized applied mathematics over pure mathematics. This trend is reflected in the naming of many academic departments as “Departments of Applied Mathematics”, likely to highlight the perceived practicality of the term “applied”. Such strategies aim to attract more students and secure funding from sources beyond the traditional realm of mathematics [1, 2]. Furthermore, a detrimental consequence for mathematical science curricula would likely arise. Many individuals outside the field of mathematics dismiss pure mathematics as irrelevant or impractical. This perspective has led to a reduction in the number of mathematical courses, even within applied mathematics programs. Instead, there is often an emphasis on promoting data science and statistics, with the assertion that these disciplines are exceptionally valuable in contemporary applications. This stands in stark contrast to the views of dedicated mathematicians, who recognize the fundamental importance of pure mathematics. At one extreme, some mathematicians label their research as applied mathematics, even when it is fundamentally pure mathematics, to appear more “practical” and “useful.” At the other extreme are those who work exclusively on the applications of mathematics in fields such as finance, industry, and biology. While their work involves the implementation of mathematical tools to address specific problems, it does not constitute research in mathematics, as it does not generate new mathematical knowledge or methods.

The authors posit that the aforementioned problems stem from divergent perspectives within the philosophy of mathematics and related inquiries, such as, What constitutes mathematics? What are the distinctions and relative importance of pure mathematics, applied mathematics, and the applications of mathematics? This paper endeavors to address these issues in detail, ultimately exploring the true nature of mathematical research.

In the existing literature, Buckley [3] and Moslehian [4] have discussed the significance of pure mathematics and its benefits to global society. They concur that mathematics is the logical and abstract study of patterns. Furthermore, the strength of pure mathematics resides in its abstraction, as this maximizes its potential versatility [3]. Applied mathematics can be conceptualized as a bridge connecting pure mathematics and real-world applications [4].

What is Mathematics?

Mathematics is often regarded as the most effective science for discovering and analyzing patterns that help us understand the world and the universe we live in [5, 6]. This broad definition naturally emerges from the historical development of mathematics and science. For undergraduate students, taking a course in the history of mathematics is essential for gaining a deeper perspective on what mathematics truly represents.

The foundations of mathematical thought can be traced back to figures like Pythagoras (c. 570-435 BC), who famously stated, “Mathematics is the way to understand the universe.” He introduced the concept that “Number is the measure of all things” and is best known for the Pythagorean Theorem:

in a right triangle, the square of the hypotenuse (the side opposite the right angle) equals the sum of the squares of the other two sides.

In the early modern period, Johannes Kepler (1571-1630) was captivated by the idea that God's design of the world and universe could only be understood through mathematics. He demonstrated that the complex movements of planets could be entirely described using three simple mathematical laws, now known as Kepler's Laws of Planetary Motion.

Galileo Galilei (1564-1642), often referred to as the "father of astronomy", "father of modern physics", and "father of modern science", famously stated: "The universe cannot be read until we have learned the language and become familiar with the characters in which it is written. It is written in the language of mathematics, and its letters are triangles, circles, and other geometrical figures, without which it is humanly impossible to comprehend a single word."

Building on the work of Kepler and Galileo, Isaac Newton (1642-1727) formulated the famous Newtonian Laws of Motion, demonstrating one of the most profound applications of mathematics in science.

Modern mathematics, however, extends far beyond calculations and deductions. It involves observing patterns, formulating definitions, proving conjectures, and estimating results. Mathematics is, at its core, the abstract science of pattern and order. While logical reasoning remains its foundation, observation and simulation often provide valuable insights into the truths it seeks to uncover.

Characteristics of Mathematical Thinking:

- Generality and universal applicability: Mathematical ideas apply across diverse disciplines and problems.
- Elegance and depth: Mathematical concepts, including theories, theorems, and proofs, are valued for their elegance, depth, and significance. Good mathematics often embodies beauty.
- Foundation of truth and certainty: Mathematics offers a standard of certainty and a foundation of truth for other scientific fields.
- Unique models of thought: Mathematics employs distinct methods such as logic, abstraction, symbolism, optimization, modeling, and the inference of universal truths from data samples.
- Beauty and power: The beauty of mathematics emerges from abstract concepts like symmetry, proof, and transformation, while its power lies in its ability to solve complex problems.

In this way, mathematics is not only a tool for understanding the universe but also a discipline that embodies both intellectual rigor and aesthetic appeal. In particular, our viewpoints are in line with [3, 4].

The Pyramid of Mathematics: Pure Mathematics, Applied Mathematics, and Applications of Mathematics

Modern mathematics can be conceptualized as a pyramid, as shown in Figure 1. At the base of the pyramid lies pure mathematics, which serves as the foundation for the entire structure. Above this is applied mathematics, and at the top are the applications of mathematics, represented as the space under the roof of the pyramid. In the following sections, we will discuss each layer in detail.

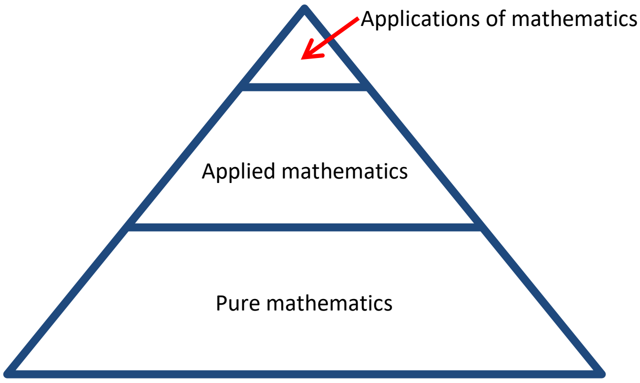

Figure 1 The pyramid of mathematics.

Pure Mathematics

Roughly speaking, pure mathematics is the branch of mathematics that focuses on the study of abstract objects and concepts [7, 8]. Although these objects and concepts are simplified models of real-world phenomena, the results of pure mathematics often turn out to be highly practical, even though this application may take years to materialize.

To remain general and adaptable to various needs, pure mathematics is not necessarily directly related to practical applications. It is possible to study abstract objects and concepts in isolation, focusing solely on their intrinsic nature without immediate concern for their real-world manifestations.

Nevertheless, the distinction between pure and applied mathematics is often blurred, as the two fields frequently overlap and influence each other. In fact, many applied mathematicians use tools and techniques from pure mathematics to construct useful and accurate models of the real world. Conversely, many ideas in pure mathematics have been inspired by phenomena in nature or social contexts.

Pure mathematics is characterized by its emphasis on generality and abstraction. These qualities are crucial because:

- General theorems and concepts lead to a deeper understanding of original ideas and results.
- Generality and abstraction can simplify thinking, leading to shorter, more elegant proofs and arguments that are easier to follow.
- Generality avoids redundancy: proving a general result is often more efficient than proving separate cases independently.

- Generality can foster connections between different branches of mathematics and even other scientific fields.

As the foundation of the mathematical pyramid, pure mathematics must be robust enough to support the higher levels of applied mathematics and its applications. The main fields of pure mathematics include:

1. Analysis: The study of changes through concepts of limits, continuity, convergence, differentiation, and integration. It forms the basis for calculus and many other mathematical fields. Two important concepts in analysis are metrics and measures. Metric is the abstract notion of distance between two objects. Measure is the abstract size (e.g. length, area, volume) of objects, so that measures are used in integration theory.
2. Algebra: The study of sets with certain (mostly, binary or unary) operations defined on them. These sets and operations are classified according to their properties, leading to concepts such as groups, rings, fields, modules, lattices, Boolean algebras, and many others. Linear algebra, as a branch of algebra, studies linear algebraic systems, linear spaces, and linear transformations. Boolean algebra studies algebraic systems consisted of a set together with two binary operations and a unary operation satisfying Boolean laws.
3. Geometry and Topology: The study of shapes and spaces, particularly transformations that act on these spaces. For example, differential geometry applies the techniques of differential and integral calculus, as well as linear algebra, to solve geometric problems. Geometry developed into topology, which studies topological spaces and continuous maps between them, focusing on how spaces are connected while ignoring precise measurements of distance and angles.
4. Discrete Mathematics: The study of discrete objects included integers, graphs, and logical statements. Number theory is the study of positive integers, using concepts such as divisibility and congruence. A fundamental result in number theory is that every positive integer has a unique prime factorization. Although number theory is a field that can easily engage the general public, as anyone can explore the properties of numbers, it is also one of the most challenging areas for research. For instance, the proof of Fermat's Last Theorem and its generalizations is notoriously difficult to understand. Graph theory is the study of objects (vertices) together with their pairwise relations (edges). Combinatorics (or combinatorial theory) is the study of the ways to combine or arrange discrete objects.

These fields are not isolated but are deeply interconnected. Moreover, many modern mathematical branches emerge at the intersection of different fields. For example:

- Algebraic geometry studies geometric objects using abstract algebra.
- Algebraic number theory applies abstract algebra to the study of positive integers.
- Analytic number theory uses complex analysis to explore the properties of positive integers.
- Algebraic graph theory studies graphs using algebraic concepts such as linear algebra and abstract algebra.

Applied Mathematics and Its Interconnection to Pure Mathematics

Applied mathematics is the branch of mathematics that develops mathematical methods and techniques to address specific problems in various fields, including science, engineering, business, computer science, and industry [9]. Generally, applied mathematics is more challenging than pure mathematics because it requires not only mathematical expertise but also specialized knowledge from the respective domain in which mathematics is applied. Without the foundational understanding provided by pure mathematics, there would be no basis for application.

It is important to distinguish applied mathematics, a field of study within mathematics, from the applications of mathematics, which will be discussed later.

Broadly speaking, applied mathematics involves adapting pure mathematical methods to solve practical problems. For instance, in the 17th century, Newton developed calculus and laid the foundations of analysis to compute the orbits of planets. Over time, calculus evolved into a field of pure mathematics, and today its applications extend far beyond its original purpose.

The British mathematician G.H. Hardy expressed a similar idea, stating that pure mathematics is, in the long run, often more useful than applied mathematics. The usefulness of a mathematical theory arises from its generality and abstractness. For example, group theory has numerous applications in chemistry, physics, mathematics, and biology precisely because of its abstract and general nature; many real-world and mathematical structures exhibit group-like properties. While the origins of the phrase "there is nothing more practical than a good theory" remain uncertain, it aptly captures the essence of this relationship between theory and application.

It is difficult to provide a comprehensive list of all the subfields within applied mathematics, but here are some of the key areas:

- Scientific computing, numerical analysis, computing science, and mathematical modeling
- Operations research
- Mathematical statistics
- Actuarial science: Applying probability, statistics, and economic theory to assess risk in fields such as insurance and finance
- Mathematical economics: The study of mathematical methods to model and analyze economic problems. Mathematical economics draws on statistics, probability,

programming, operations research, game theory, and more. Note that mathematical economics differs from financial mathematics.

- Applied mathematics in other disciplines: Including business, engineering, physics, chemistry, psychology, biology, and mathematical physics.

Applications of Mathematics

The final section at the top of our mathematical pyramid represents the applications of mathematics in various fields of science, such as engineering, computer science, biology, and others. However, it is important to note that this is not a distinct field of mathematics; rather, it involves the use of mathematical results and techniques within other disciplines.

Although the distinction between applied mathematics and the applications of mathematics can sometimes be unclear, a simple way to differentiate the two is as follows: Applied mathematics is a branch of mathematics that develops mathematical methods and techniques, often derived from pure mathematics, to solve specific problems in various fields. These problems typically cannot be addressed by existing methods within the given field. On the other hand, the applications of mathematics refer to the use of known mathematical results whether from pure or applied mathematics to solve practical problems. This is not a form of mathematics itself, but rather a process that belongs to other disciplines.

Contemporary Real-World Applications of Mathematics

In this section, we discuss the contemporary role of mathematics. Before that, it is essential to emphasize that pure mathematics cannot be separated from real-world applications. Many mathematical discoveries that once appeared to be purely abstract or even useless in their time have since proven to be of immense practical value.

Example 1: Negative/complex numbers and scientific disciplines

Negative and complex numbers, discovered in antiquity, were considered nonsensical and irrelevant until the 15th century. Today, however, we recognize the importance of negative numbers in nearly every field of mathematics, both pure and applied, as well as in a wide array of scientific disciplines. Complex numbers have countless applications, including electrical engineering, quantum computing, and many other fields.

Example 2: Euclidean geometry and GPS systems

Around 2000 years ago, Euclid developed planar geometry based on the following five postulates:

1. A straight line can be drawn between any two points.
2. A segment of a straight line can be extended indefinitely.
3. A circle can be drawn with any given radius and arbitrary center.
4. All right angles are equal.

5. If three straight lines (L_1, L_2 and L_3) form angles A and B less than 180°, L_1 and L_2 will meet on the side of the angles when extended indefinitely.

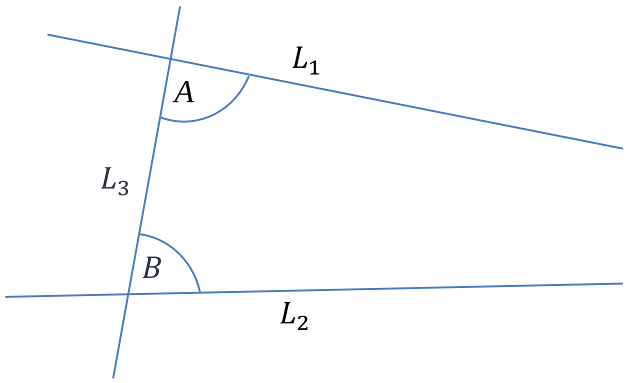

Figure 2 The fifth postulate of Euclid.

For nearly two millennia, mathematicians attempted to deduce the fifth postulate from the others. In the 19th century, Lobachevsky and Bolyai proved the independence of the fifth axiom, a result that was initially considered purely theoretical. Its real-world application came later when Einstein utilized this concept in his theory of general relativity. Today, general relativity is fundamental to technologies such as GPS systems.

Example 3: Graph theory and the board game chess

Although graph theory is formulated in a highly abstract and general way, it has widespread applications in diverse fields such as chemistry, biology, and computer science. In graph theory, the famous four-color-map problem was successfully solved in 1976 via computer assistance [10]. Recall that chess is a strategic board game played on 8x8 chessboard. Other interesting problems in graph theory, related to chess, are the knights tour problem and the eight-queen problem. The first problem seeks a path of knight's moves on the chessboard so that the knight visits every square exactly once.

Example 4: Radon, Wavelet, and Fourier transforms

The Radon transform, introduced in 1917 as an integral transform in the context of harmonic analysis, was initially a topic in pure mathematics. However, in the 1960s, it found significant application in tomography and CT scanning technology, leading to groundbreaking advances in medical imaging. This application of pure mathematics to medicine ultimately contributed to a Nobel Prize in Medicine. Similarly, wavelet and Fourier transforms, originally conceived in pure mathematics, now play crucial roles in fields such as computer graphics and medical technology, including in MRA scans, blood pressure monitors, and diabetes management devices.

Example 5: Mathematical logic and the Turing machine

Alan Turing's theory of computability, rooted in mathematical logic, forms the foundation of modern computer science. It underpins all digital technologies, including personal computers, smartphones, and various other electronic devices.

Example 6: Number theory and computer security

Historically, number theory was often dismissed as abstract and without practical value. However, the study of prime numbers is now essential in the design of computer security systems, including those used for online shopping and credit card encryption.

Example 7: The Laplace equation and physical phenomena

The Laplace equation, a key partial differential equation in mathematics, was originally studied from a pure mathematical perspective. Over time, it became clear that this equation governs a variety of phenomena, including those in astronomy, electromagnetism, fluid dynamics, and heat distribution. A deep understanding of the equation thus provides insight into a wide range of physical processes.

Example 8: Boolean algebra and digital circuits

Boolean algebra is a branch of algebra in which the values of (Boolean) variables are truth values, *true* or *false*, usually denoted by *1* and *0*, respectively. Operations for such variables are Boolean (logical) operations, such as negation (*not*), conjunction (*and*), and disjunction (*or*). Boolean algebra is strongly connected to Boolean logic and digital circuits. Indeed, digital circuits are modeled via logic gates (e.g. *NOT, AND, OR* gates), and the implements of such logic gates are done by Boolean operations. The Boolean values *1* and *0* electronically represent the supply voltage and the ground, respectively. See more information about the theory and applications of Boolean algebras in [11].

Example 9: Quaternions and color image processing

Quaternions, first introduced by W.R. Hamilton in 1943, is a four-dimensional algebraic system together with the operations of addition and multiplication satisfying certain laws. In particular, the quaternion number system extends the complex number system. In mechanics, quaternions can be applied to describe motions in three-dimensional space. The multiplication of two quaternions physically describes the composition of two rotations. Recall that in color image processing, a color image can be described by a vector or a matrix. In RGB (red-green-blue) color model, the color information of each pixel can be represented via quaternions. The addition between the two quaternions represents the superposition of their light beams of different colors. See more information about the theory and applications of quaternion algebras in [12-14]. Moreover, in the last decade, deep learning models require certain algebraic number systems like quaternions as fundamental theory; see the survey [15].

Example 10: Markov process and the AI AlphaGo

Let us recall that Go is an abstract strategic board game between two players on the 19x19 board. The goal of this game is to enclose more territory than the opponent. AlphaGo, developed successfully in 2015 by DeepMind Google team, is an artificial intelligence program playing Go. AlphaGo has proved to be much stronger than any human player; this AI defeated Lee Sedol, the world Go champion at that time, 4:1 in the DeepMind Challenge Match 2016. From mathematical perspective, Go is an alternating Markov game. The computer program AlphaGo is a neural network in which the input is a board position, and the outputs are a sequence of moves and an evaluation of the given position (*value*). This neural network is a real-valued function parametrized by a real vector, trained by reinforcement learning, so that a win increases the probability of contributing moves. To train these neurons, one applies a certain iterative process using a fixed-point theorem for contraction maps on a complete metric space of *value functions* for any alternating Markov game. See more information in [16].

Example 11: Differential geometry and transport problems

The field differential geometry utilizes calculus and linear algebra to study the geometry of smooth geometrical objects. In the classical case, when traveling from point P to a point Q in the Euclidean space, the distance and hence the time is the same as when traveling from Q to P. However, in engineering, transport and control problems, etc., there are external forces in effect on the traveling object, like winds, magnetic/gravitational fields, etc. Thus, the time minimal paths and the travel times are different according to the direction relative to the external forces. This observation leads to the study of various notions of distances and metrics, e.g. Randers metrics and Finsler metrics. In particular, the geometry on a slope of a mountain can be investigated through a Finsler metric. See more information in [17-19].

Example 12: Topology and data science

Topology, the branch of mathematics concerned with the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing, originated as a field within pure mathematics. Today, topology has evolved, and the emerging field of computational topology applies topological concepts, such as homotopy and homology, to analyze complex data sets in fields like data science and machine learning.

By examining these examples, along with many others that could not be included here, we can conclude that pure mathematics is fundamental to all branches of mathematics and applied science. No meaningful research in applied mathematics can be conducted without a solid foundation in pure mathematics. Mathematics functions as a pyramid, with pure mathematics forming its base. The higher levels of this pyramid cannot stand without a wide and sturdy foundation. A tall pyramid with a narrow base will inevitably collapse under pressure. Therefore, while applied mathematics and the applications of mathematics are vital and valuable, it is crucial to have a thorough grounding in pure mathematics before venturing into these areas. Without pure mathematics, there is nothing to apply.

Research in Mathematics

Having explored what mathematics is and its pyramid structure, we now turn to the final section of our discussion: research in mathematics.

From the preceding sections, it should be clear that research in mathematics involves either pure mathematics or applied mathematics. While the applications of mathematics can serve as valuable exercises, seminar topics for graduate students, or consulting opportunities for companies and researchers in other fields, they do not constitute mathematical research unless they involve the development of new mathematical concepts or techniques. Applications of mathematics alone, without contributing to the advancement of mathematical theory, are not part of the research process.

In mathematics, it is uncommon to say, "I am computing this or that." Rather, we would say, "I am exploring this or that." This reflects the essence of mathematical research, which is rooted in reflection and analysis of abstract ideas. Unlike other disciplines where practical work or experiments are central, mathematics is largely an intellectual pursuit, where the aim is to probe deeper into the relationships and structures that underpin mathematical theory.

One of the primary motivations for conducting mathematical research is the desire to understand the beauty and significance of abstract patterns in nature. We engage in research to uncover the non-trivial aspects of complex patterns that often go unnoticed in our everyday experiences. For instance, if you study a Buddhist mandala, the decorations of the Alhambra in Granada, Spain, or listen repeatedly to a fugue by Bach, you will begin to identify patterns that were previously hidden. This process is like mathematical research. However, while identifying patterns in art or music relies primarily on our intuition, research in mathematics demands an additional "sense," one that is cultivated through the rigorous study of pure mathematics specifically, theorems and proofs.

Unfortunately, many educators omit proofs while teaching mathematics, which is one of the greatest missteps in mathematical education. Even if some details are skipped or left as homework assignments, it is crucial for students to understand the core idea behind each proof. A proficient mathematician is someone who can at least explain the central idea behind every result in their field. Engaging in research without this foundational understanding is akin to attempting to read a book without knowing the alphabet.

To conduct research in mathematics, one requires a solid mathematical background. This background is typically acquired through standard undergraduate or graduate courses, such as calculus, differential equations, linear algebra, abstract algebra, real analysis, and topology. Once one has mastered these foundational concepts and techniques, it becomes easier to develop new theories or methods.

Example 13: Poincaré conjecture

A famous research topic in the field of geometric topology is the *Poincaré conjecture*, a mathematical problem stating that *every closed, simply connected, three-dimensional manifold is a sphere*. Here a manifold is a mathematical shape, like a sphere, cube, or blob, a closed manifold is a manifold without boundaries or holes and a simply connected manifold is a manifold where every loop can be continuously tightened to a point. The result is about the topological equivalence of two manifolds, that is, the manifolds are homeomorphic in the sense that two sets can be mapped one-to-one and continuously. Originally, the conjecture was formulated by H. Poincaré in 1904 and proved in 2002 by the Russian mathematician G. Perelman using the notion of Ricci flow. It took almost a century to understand it. This is the power of pure mathematics used for studying applications. See more information in [20].

Example 14: Control theory, matrix equations, and iterative algorithms

The field of control theory, dealing with applied mathematics and optimal control, investigates the behavior of dynamical systems in physics and engineering. The stability and structural problems in this field lead to the study of certain linear matrix equations, such as Lyapunov, Sylvester, and Stein equations. A direct algebraic method to find the solutions of such matrix equations can be devised using notions of traditional linear algebra. However, when the sizes of associated matrices are large, it requires the use of computers with large memory to calculate an exact solution. Thus, for practical purposes, it is convenient to develop numerical algorithms for solving such matrix equations for exact or well-approximate solutions. In the last two decades, many researchers utilized various techniques to derive iterative procedures for solving such equations; see more information in [21-23] and references therein.

Example 15: Electrical circuits, chaos theory, and differential equations

Another example is the subject of chaos theory, which is dealing with applied mathematics, physics, and electrical engineering. The subject studies nonlinear dynamical systems having a non-periodic oscillation of waveforms. A significant development of this theory is the discovery of Chua's electrical circuit; see e.g. [24]. Using physical laws such as Ohm's law and Kirchhoff voltage/current laws, we can describe the Chua's chaotic phenomenon by a system of ordinary differential equations. See more information in [25].

Research in pure mathematics yields tremendous societal benefits. Gaining an understanding of general and abstract concepts in mathematics is akin to teaching a child to count. Initially, the child counts tangible objects such as toys or fruits but eventually grasps the abstract concept of counting itself, which can then be applied universally. Similarly, in mathematics, we begin with concrete problems and gradually develop abstract concepts that can be applied to a wide range of scenarios. This process of abstraction is the strength of mathematics, as it maximizes the potential usefulness of mathematical theories across different domains (as demonstrated by the examples provided earlier).

We conclude that the distinction between pure and applied mathematics is somewhat artificial. Many important areas of modern pure mathematical research have significant implications for practical applications. Conversely, when engaged in applied mathematics research, we inevitably encounter abstract mathematical concepts such as properties of special matrices or algebraic groups that arise naturally from the study of real-world problems.

Finally, mathematical research not only contributes to the broader world but also enhances the quality of teaching and fosters deeper insights into theorems and their proofs. These proofs are, after all, the most valuable teaching resource in mathematics.

Conclusions

Nowadays, mathematics is an essential subject at every level of education, as well as in the research community. Indeed, there are two major branches of mathematics, namely, pure mathematics and applied mathematics. Pure mathematics focuses on the study of patterns for abstract objects and concepts, e.g. the operations between numbers and the shape of geometric objects. Applied mathematics uses mathematical methods and techniques, derived from pure mathematics, to solve specific problems in various fields. The characterization of mathematics is the generality and universality, the elegance and depth, the foundation of truth and certainty, unique model of thoughts, and the beauty and power. It turns out that mathematics plays an important role in every subject area of science, engineering, computer science, economics, social science, etc. Mathematics is beyond almost everything we face in daily life, such as smart electronics devices, not even board games like chess and Go. Researches in mathematics thus mean researches in pure or applied mathematics, that is, the development of new mathematical concepts/theory or methods. To do that, one must have adequate background from standard courses in mathematics. Applications of mathematics to other disciplines, without contributing new mathematical theory/ methods, are not part of mathematics research, but rather a research process in others. In the last decade, research in mathematics grow rapidly and there are much interconnections between different subfields of mathematics as well as other disciplines.

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