บทความวิจัย

เฟสแม่เหล็กในแบบจำลองไอซ์ซิงค์พันธะสุ่มแบบปรับปรุง

ลือชา ตั้งชนะชัยอนันต์ และ ฉัตรชัย ศรีนิติวรวงศ์*

บทคัดย่อ

บทความนี้ได้ศึกษาเฟสแม่เหล็กสำหรับแบบจำลองไอซ์ซิงค์พันธะแบบสุ่มแบบปรับปรุง แบบจำลองนี้มีพื้นฐานมาจากแบบจำลองไอซ์ซิงค์ที่มีพันธะแบบสุ่ม โดยค่าอันตรกิริยาแม่เหล็กมีการกระจายตัวแบบเการ์เชียน โดยความกว้างของการกระจายตัวแบบเการ์เชียน (J) เป็นค่าที่ปรับได้ ค่าความกว้างของการกระจายตัวสัมพันธ์กับองศาของการไร้ระเบียบของระบบ บทความนี้จะคำนวณค่าแมกนิไตเซชัน ค่าแก้วสปินแมกนิไตเซชัน และ บินเดอร์คิวมูแลนเพื่อระบุเฟสแม่เหล็ก พบว่ามีเฟสปรากฏสามเฟสบนแผนภาพ J-T เมื่อ T คืออุณหภูมิ คือ (1) เฟสแม่เหล็กเฟอร์โร (2) เฟสแก้วสปิน และ (3) เฟสแม่เหล็กพารา

คำสำคัญ: แบบจำลองไอซ์ซิงค์ เฟสแก้วสปิน

ภาควิชาฟิสิกส์ คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

*ผู้นิพนธ์ประสานงาน, e-mail: Chatchai.s@chula.ac.th

Magnetic Phases in Modified Random Bond Ising Model

Luecha Tungchanachaianun and Chatchai Srinitiwarawong*

ABSTRACT

The magnetic phase diagram of a modified random-bond Ising model has been investigated. The model is based on the random-bond Ising model in which the magnetic interactions are distributed in Gaussian form. The width of the Gaussian distribution (J) of the interaction has been varied. The variance of the distribution width corresponds to the degree of disorder in the system. By using magnetization, spin glass magnetization and binder cumulant, three ordered phases appear in the *J-T* phase diagram where T is temperature: (1) ferromagnetic (FM) phase; (2) spin glass (SG) phase; (3) paramagnetic (PM) phase.

Keywords: Ising model, spin glass phase

Department of Physics, Faculty of Science, Chulalongkorn University
*Corresponding author, e-mail: Chatchai.s@chula.ac.th

Introduction

For the magnetic film on homogeneous single substrate, the previous experimental results show that the important parameters such as magnetization, heat capacity, magnetic susceptibility, etc., are similar to the values that calculated from the Ising model. But in the homogenous binary mixture, it will become thermodynamically unstable which cause by the disorder in the system [1, 2]. One of the suitable models to describe this system is random-bond Ising model (RBIM). In the RBIM, the interactions are in the form of probability distribution [3], e.g., Gaussian [4], uniform [3], bimodal [5], etc. The values from the experimental result are in agreement with that of numerical simulations for the system of quenched disorder caused by random-bond defects which happen in the homogenous binary mixture [6-11].

Another model is Edward-Anderson (EA) spin glass model [12] which base on binary mixture that an array of spins of one material arranged at random in the matrix of another material. The interactions between spins are Gaussian distribution with zero mean therefore it cannot be classified as ferro-or anti-ferromagnetism. But there is a ground state in which the spins aligned in certain directions which appear to be random for each spin. The RBIM and EA spin glass models both are developed from Ising model by changing the interaction from single value to multi-value with the probability distribution function that constrained the value of interaction in certain range.

In the Ising model, the magnetic interactions are a single value while in the random-bond Ising model, interactions are distributed according to the distribution rules. When the interactions distributed with zero mean, the domains of ferromagnetic and antiferromagnetic occur equally. When we look at macroscopic scale, the magnetization of the system is zero. But if we look at microscopic scale, the local magnetization is not zero which is the property of spin glass. In this work, we will focus on the random-bond Ising model in which the distribution of magnetic interactions is Gaussian distribution. By using the hypothesis that adding another material will cause the system to become disorder. As the result, the width of the interaction became wider. Since the RBIM's interactions always more than zero while EA spin glass's interaction can be both positive and negative with zero average, there should be cross-over regimes between ferromagnetic random-bond and possible complex phases such as spin glass phase as the width of the interaction distribution is varied.

Model

We investigate phase transition of the modified random-bond Ising model in which the magnetic interaction is distributed according to Gaussian distribution. The Gaussian distribution in the spin glass model in the previous reference [4] has zero mean and distributed equally in the negative and positive regions. In random bond Ising model, the magnetic interaction is uniformly distributed in positive region only. However, in this model the magnetic interaction is distributed in the Gaussian form with positive mean and the width of the interaction is varied. The model consist of 2-dimensional square lattice Ising spin $S_i = \pm 1$. The Hamiltonian of the system can be written as

$$H = -\sum_{\langle ij \rangle} J_{ij} S_i S_{ij},$$

while the interactions J_{ij} are distributed according to

$$p(J_{ij}) = [(2\pi)^{1/2} J]^{-1} exp[-(J_{ij} - J_0)^2 / 2J^2],$$

and the interactions are accounted only the nearest neighbor sites. Parameter J indicates the width of interaction's distribution therefore it directly related to the degree of disorder in the system. Parameter J_0 is the center of distribution as shown in figures 1. In this model, the interactions can be both positive and negative depending on the width of the interaction. At low disorder ($J \to 0$), the interactions distribute in the positive region only. As the result, the system should behave like the Ising model. At high disorder ($J >> 0$), the interactions distribute in both positive and negative regions. As the result, the system should behave like spin glass model. There should be a cross over between Ising phase and spin glass phase as the width of the interaction is increased.

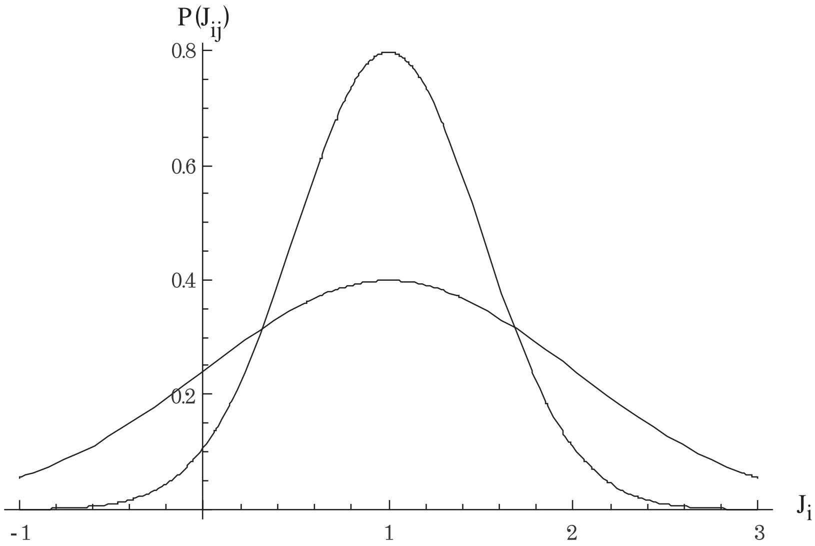

Figure 1 Gaussian distribution interaction with high and low disorder.

The center of interaction distribution has been set to 1 and the value of *J* is start from 0 to 2, while the temperatures are varied from 1 to 4. Unit of all value are in unit of energy. The number has been used in order to compare the strength of interaction with thermal energy.

Results

We use Monte Carlo simulation method to obtain the result. The simulations are performed in heat-bath MC method [13] with the periodic boundary condition. The results are divided in to two parts. In the first, the phase diagram has been drawn according to average magnetization and spin glass magnetization parameters. The second part, some of the transition points have been obtained from binder cumulant.

Phase Diagram-

The phase diagram can be obtained with 2 parameters [14], average magnetization (m) and mean-square disorder local moment or spin glass magnetization (q). Both parameters can be defined as [14, 17],

$$m = \frac{\sum S_i^{\alpha}}{N}$$

$$q = \frac{S_i^{\alpha} S_i^{\beta}}{N}$$

Magnetic phase of the system can be categorized as the following [15]

1. paramagnetic if both m and q are zero.
2. ferromagnetic if both m and q are non-zero.
3. spin glass if q is non-zero but m is zero.

The average magnetization can be used to identify the ferromagnetic or paramagnetic in the macroscopic scale. But spin glass phase and paramagnetic phase both have zero average magnetization, so only average magnetization is not enough to distinguish these phases. The mean-square disorder local moment or spin glass magnetization is used to measure the magnetic order of the system. In macroscopic scale, spin glass phase behaves similar to paramagnetic phase. However in microscopic scale, the magnetic domains occur in spin glass phase due to the interactions' distribution therefore the spin glass phase have zero average magnetization but non-zero spin glass magnetization. The results from the simulation of average magnetization are shown in figures 2 and 3 and spin glass magnetization are shown in figures 4 and 5. The phase diagram can be determined by combine these diagrams together as shown in figure 6.

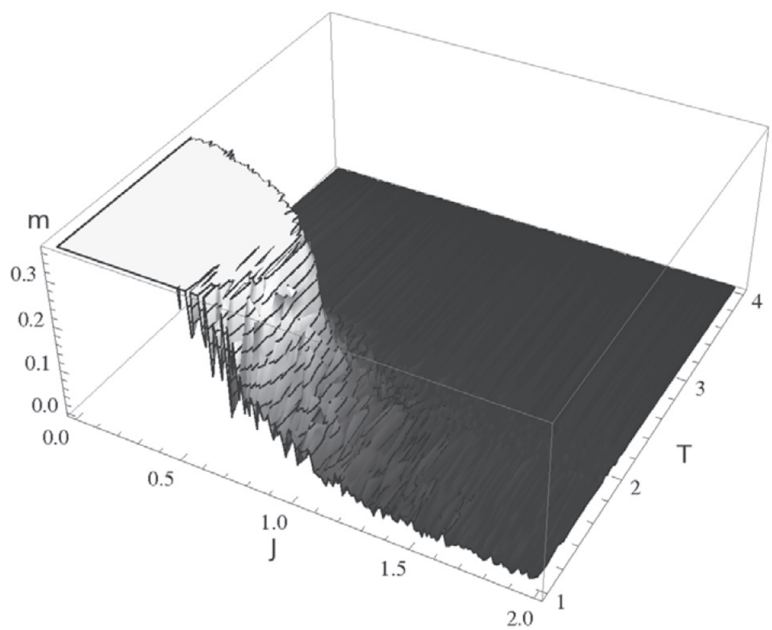

Figure 2 m–J–T phase diagram

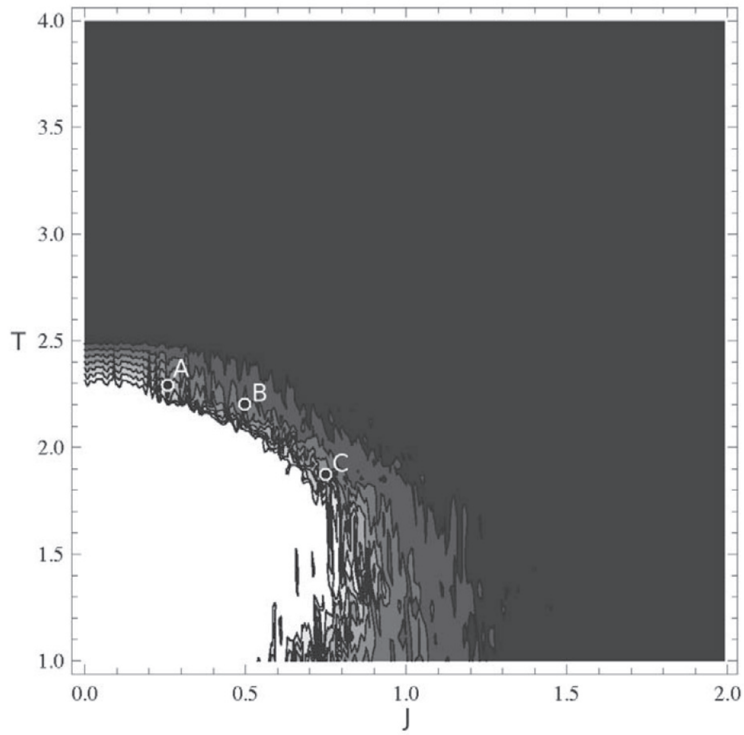

Figure 3 m–J–T phase diagram

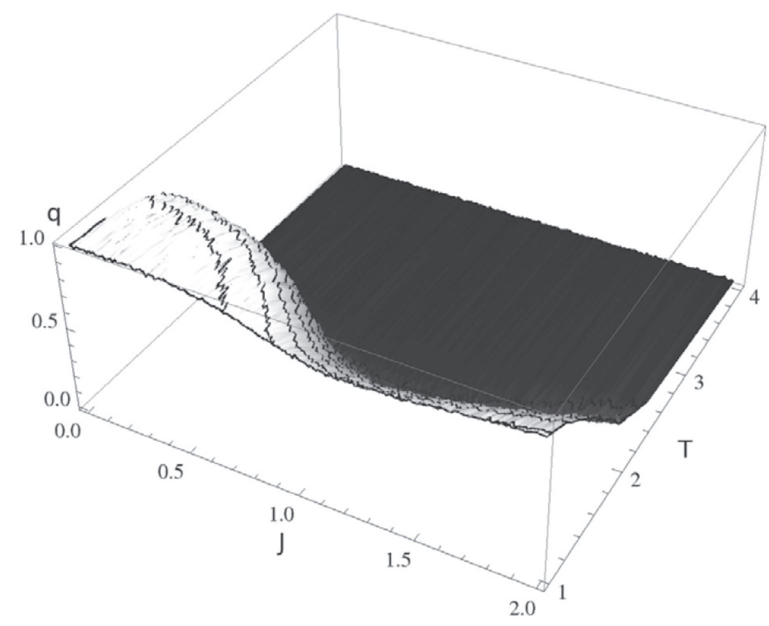

Figure 4 q–J–T phase diagram

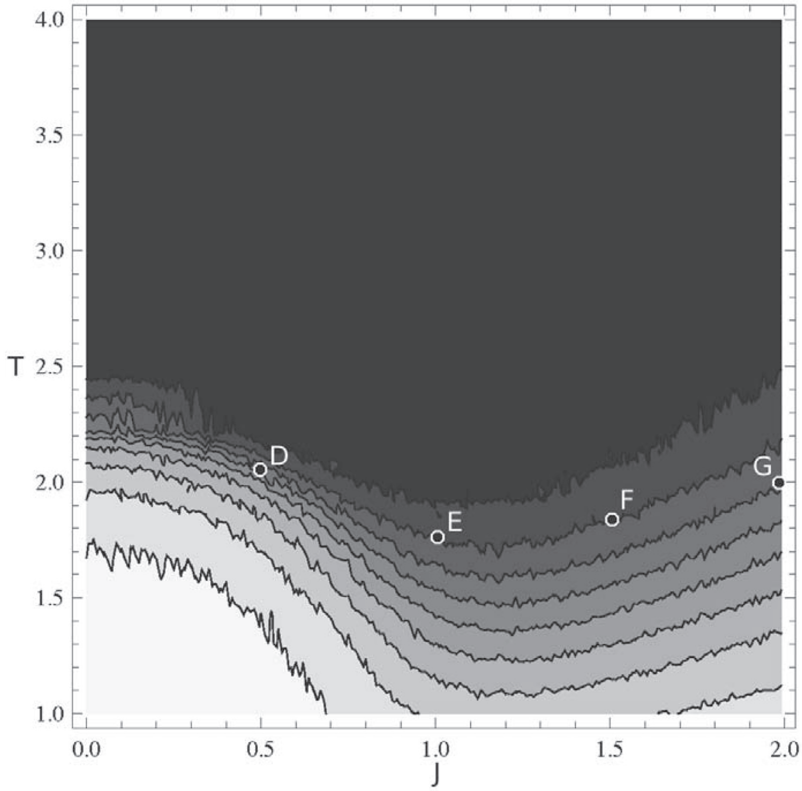

Figure 5 q–J–T phase diagram

Binder cumulant

The Binder cumulants which is used in this work are Binder cumulant for magnetization [17] and spin glass Binder cumulant [16, 18]. The Binder cumulant for magnetization is defined by

$$U = 1 - \frac{\langle m^4 \rangle}{3\langle m^2 \rangle^2}$$

and the spin glass Binder cumulant is defined by

$$g = \frac{1}{2}(3 - \frac{\langle q^4 \rangle}{\langle q^2 \rangle^2}).$$

At critical temperature, the Binder cumulants are independent of the size of the system, as a result, the Binder cumulants should be the same value. The critical temperature which the transition occurs can be determined by the intersection of binder cumulants. We simulate the Binder cumulant with the system size of 10 x 10, 20 x 20, 30 x 30, 40 x 40 and 50 x 50. The binder cumulants of the system have been calculated by changing the temperature at certain value of parameter *J*. It is expected that with higher value of *J* the intersection point in Binder cumulant for magnetization disappear. On the other hand with decreasing of value of *J* the intersection points in the spin glass Binder cumulant should be at the same position as the intersection point in Binder cumulant for magnetization.

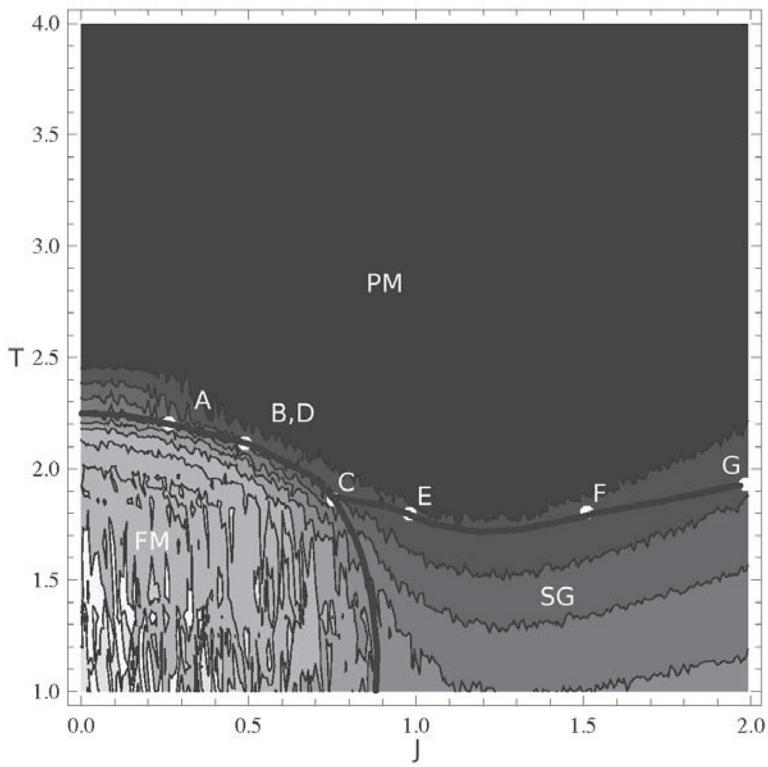

Figure 6 Phase diagram of the system.

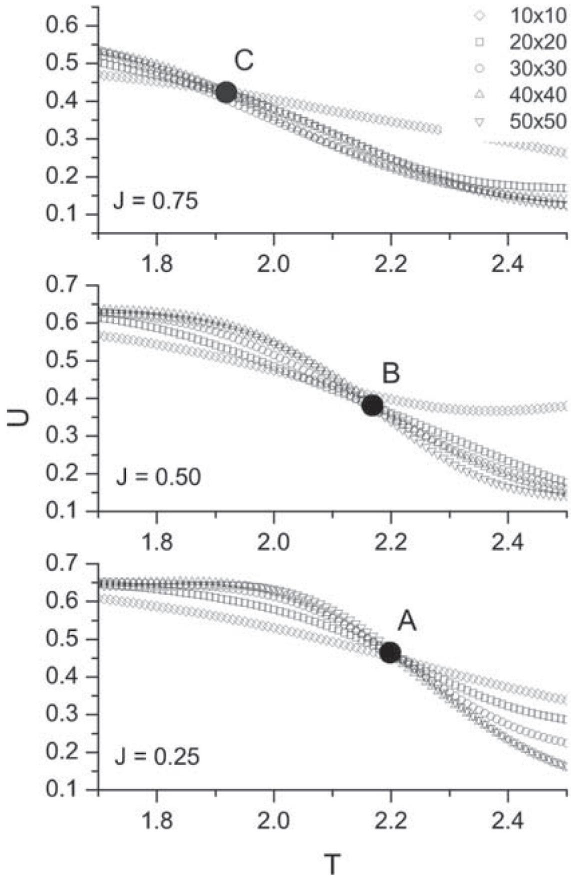

Figure 7 Magnetization Binder cumulant as a function of temperature at J = 0.25, J = 0.50, J = 0.75 at difference system size.

The transition from ferromagnetic phase to paramagnetic phase can be observed from the intersection points in figures 7 and the transition from spin glass phase to paramagnetic phase can be observed from the intersection points in figures 8. Points A, B and C are transition points according to Binder cumulant of magnetization, U from figure 7. Points D, E, F and G are the transition points according to the spin glass Binder cumulant, g from figure 8. The results from both U and g can be used to identify the transition points rather accurately. As it can be seen from figure 6 that the transition lines, bold black line, between three phases have been drawn based on the position of the transition points obtained from the Binder cumulants. It has been seen that we are not able to obtain the transition points between SG phase and FM phase. However the transition line that separate FM phase and SG phase has been drawn according to the value of magnetization, m from figures 2 and 3. Note that points B and C are not identical but the position of these two points is close to each other.

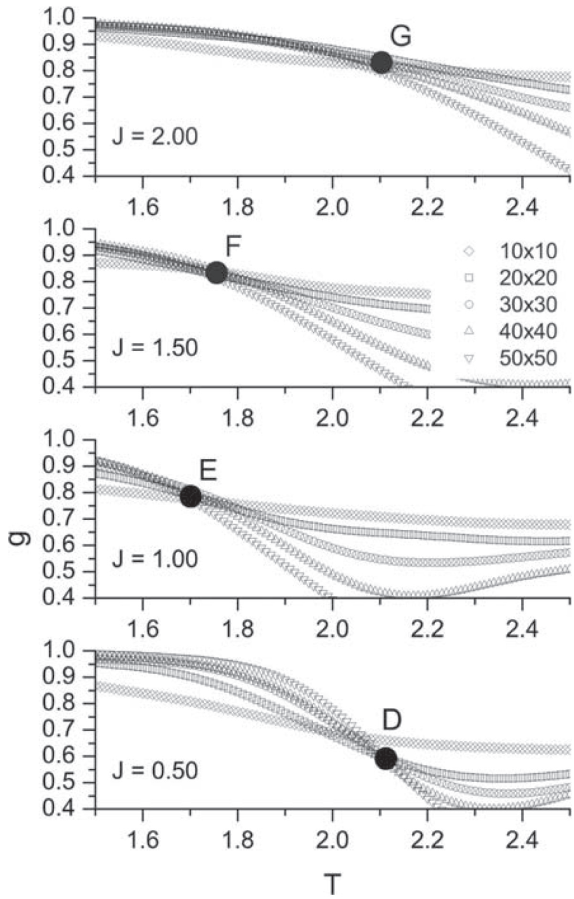

Figure 8 Spin glass Binder cumulant as a function of temperature at J = 0.50, J = 1.00, J = 1.50, J = 2.00 at difference size.

The existence of three phases can be explained based on the shape of the distribution of the magnetic interactions. At zero J the system behaves like a normal Ising model. At small value of J, the value of almost all of the magnetic interactions are positive hence the system exhibits the transition from paramagnetic phase to ferromagnetic phase as temperature decreases. At high value of J the value of magnetic interactions between pair of spins can be both positive and negative. This leads to the formation of ferromagnetic domains coupled together with antiferromagnetic interaction. This results in the vanishing of total magnetization m even at low temperature as can be seen from figure 2 and 3. However this phase is different from the paramagnetic phase in which the finite value of spin glass magnetization q occurs.

Conclusion

We have studied the phase transition in modified random-bond Ising model. The modified random-bond Ising model is base on 2 dimensional Ising model with interactions that distributed in the form of Gaussian distribution. The hypothesis is that adding another material will cause the system become disordered, as the result the width of the interaction become wider. Phase diagram can be obtained by using the diagrams of average magnetization, mean-square disorder average local moment and the Binder cumulants.

References

1. Fan, W., Jungho, K., and Kim, Y. J. 2007. Reentrant Spin Glass Transition in $LuFe_2O_4$. http://xxx.lanl.gov/abs/0712.1975. 10 March 2008.
2. Katzgraber, H. G. Herisson, D., Oesth, M., Nordblad, P., Ito, A., and Katori, H. A. 2007. Finite Versus Zero-Temperature Hysteretic Behavior of Spin Glasses: Experiment and Theory. *Physical Review B* 76(9): 092408-092412.
3. Picco, M., Honecker, A., and Pupjol, P. 2006. Strong Disorder Fixed Points in the Two-Dimensional Random-Bond Ising Model. *Journal of Statistical Mechanics: Theory and Experiment* 4: P09006.
4. Morgenstern, I. 1981. Numerically Exact Solvable Random-Bond Ising Model. *Zeitschrift fur Physik B Condensed Matter* 41(2): 147-151.
5. Fytas, N. G., and Malakis, A. 2008. Phase Diagram of the 3D Bimodal Random-Field Ising Model. *The European Physical Journal* B 61: 111-120.
6. Cugliandolo, L. F., and Kurchan, J. 1993. Analytical Solution of the Off-Equilibrium Dynamics of a Long-Range Spin-Glass Model. *Physical Review Letters* 71(1): 173-176.
7. Franzand, S., and Mezard, M. 1994. Off-Equilibrium Glassy Dynamics: A Simple Case. *EPL (Europhysics Letters)* 26: 209-214.
8. Baldassarri, A., Cugliandolo, L. F., Kurchanand, J., and Parisi, G. 1995. On the Out-Of-Equilibrium Order Parameter in Long-Range Spin-Glasses. *Journal of Physics A: Mathematical and General* 28: 1831-1845.
9. Marinari, E., Parisi, G., Ricci-Tersenghi, F., and Ruiz-Lorenzo, J. J. 1998. Relaxation, Closing Probabilities and Transition from Oscillatory to Chaotic Attractors in Asymmetric Neural Networks. *Journal of Physics A: Mathematical and General* 31: 2611-2621.
10. Franz, S., and Rieger, H. 1995. Fluctuation-Dissipation Ratio in Three-Dimensional Spin Glasses. *Journal of Statistical Physics* 79: 749-758.
11. Franz, S., Mezard, M., Parisi, G., and Peliti, L. 1999. Measuring Equilibrium Properties in

Aging Systems. *Physical Review Letters* 81: 1758-1761.

12. Edwards, S. F., and Anderson, P. W. 1975. Theory of Spin Glasses. *Journal of Physics F-Metal Physics* 5: 965-974.
13. Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., and Teller, A. H. 1953. Equation of State Calculations by Fast Computing Machines. *Journal of Chemical Physics* 21 (6): 1087-1092.
14. Fischer, K. H. 1975. Static Properties of Spin Glasses. *Physical Review Letters* 34: 1438-1441.
15. Sherrington, D., and Kirkpatrick, S. 1975. Solvable Model of a Spin-Glass. *Physical Review Letters* 35: 1792-1796.
16. Houdayer, J., and Hartmann, A. K. 2004. Low-Temperature Behavior of Two-Dimensional Gaussian Ising Spin Glasses. *Physical Review B* 70: 014418.
17. Binder,K.1981. Finite Size Scaling Analysis of Ising Model Block Function. *Zeitschrift fur Physik B Condensed Matter* 43: 119-140.
18. Bhatt, R. N., and Young, A. P. 1988. Numerical Studies of Ising Spin Glasses in Two, Three, and Four Dimensions. *Physical Review B* 37: 5606-5614.
19. Schins, A. G., Arts, A. F., and de Wijn, H. W. 1993. Domian Growth by Aging in Nonequilibrium Two-Dimensional Random Ising Systems. *Physical Review Letters* 70: 2340-2343.
20. Niidera, S., Abiko, S., and Matsubara, F. 2005. Phase Diagram of a Dilute Ferromagnetic with Antiferromagnetic Next-Nearest-Neighbour Interactions. *Physical Review B* 72: 214402.

ได้รับบทความวันที่ 17 มีนาคม 2553
ยอมรับตีพิมพ์วันที่ 30 เมษายน 2553