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ปีที่ 11 ฉบับที่ 1 เดือน มกราคม – มิถุนายน พ.ศ. 2559

Bayesian Spatio-Temporal Time Series Model for Rice and Cassava Yields in Thailand

Panudet Saengseedam and Nantachai Kantanantha*
Department of Industrial Engineering, Faculty of Engineering, Kasetsart University,
Bangkok, Thailand (phone: 662-579-8610)
*Corresponding author: Email: nantachai.k@ku.ac.th

ABSTRACT

A linear mixed model (LMM) with trend and spatial effects to forecast rice and cassava yields in Thailand is proposed. It is a modification of our previous model, which was a multivariate conditional auto regressive model (MCAR) for spatial time series data without trend. An MCAR is assumed to account for the spatial effects and a linear trend is assumed for temporal effects. A Bayesian method is adopted for parameter estimation via Gibbs sampling in Markov chain Monte Carlo (MCMC). The model is applied to the monthly spatio-temporal rice and cassava yield data, which have been extracted from the Office of Agricultural Economics, Ministry of Agriculture and Cooperatives of Thailand. Using the mean absolute error criterion (MAE), the results show that the proposed model has a better performance in most provinces in the fitting part, and all provinces in the validation part compared to the exponential smoothing (ES) with trend (Holt ES) and the MCAR from our previous study.

Keyword: Bayesian linear mixed models, Multivariate conditional auto regressive model (MCAR), Rice and cassava yields, Spatio-temporal data, Time series data

1. Introduction

Spatio-temporal time series models arise when data are collected across time as well as space. Spatio-temporal time series data can be found in various applications such as agriculture, climatology, ecology, geology, economics, and geography. They are usually collected in each area at regular intervals over a period of time. Thus the data analysis has to take account of the spatial correlations across the areas and the temporal correlations within each area. The Office of Agricultural Economics, an organization under the Ministry of Agriculture and Cooperatives of the Kingdom of Thailand, produced an annual report of some common agricultural product yields such as rice, rubber, cassava, and sugar cane, in each province of Thailand [1]. Those product yields and the study of forecasting motivated us to investigate and develop a proper forecasting model, which would be an important tool for production planning.

There have been a large number of models for time series data. Naturally, most time series in agriculture are not at all stationary. Instead they exhibit various kinds of trend, such as linear, quadratic, or exponential trends. Reference [2] found that the most common trend of rice yield in China during the years 1979-2009 is linear growth. Reference [3] presented the linear trend for cassava yield in Rwanda for the period 2000-2010. Reference [4] proposed ARIMA models to forecast boro rice production in Bangladesh and reference [5] proposed a forecasting model that can detect trend, seasonality, auto regression and outliers for vegetable prices in Thailand. For spatial data, a conditional auto regressive model (CAR) first introduced by [6] is one of the common approaches. Reference [7], extending the model of [6], proposed empirical Bayesian methods building from Poisson regression with random intercepts defined with CAR spatial correlations. Reference [8] extended the models of [7] to a full Bayesian setting for mapping the risk from a disease. Reference [9] used a Bayesian statistical model to forecast part demand time series data for Sun Microsystems, Inc.

For spatio-temporal time series data, reference [10], using a geo-statistical approach to analyze yearly data, studied the spatial and temporal variability of attributes related to the yield and quality of durum wheat production. Based on Bayesian linear mixed models with CAR spatial effects, reference [11] presented spatial time series models for rice yield in Thailand. Reference [12] proposed a linear mixed model (LMM) with spatial effects to forecast rice and cassava yields at the same time in Thailand. A multivariate conditional auto regressive (MCAR) model is assumed to present the spatial effects.

Most models for spatio-temporal time series data are based on generalized linear mixed models (GLMMs). In this paper, we apply LMM, a special case of the GLMMs, to model the agricultural product yields which are continuous data. LMM is commonly used when dealing with correlated data, due to the repeated measurements of each subject over time [13]. LMM allows fixed effects and spatial effects to be included. Recently, for complex models, the Bayesian approach is becoming increasingly popular as techniques for parameter estimation due to its extreme flexibility. Consequently, it is adopted for parameter inference in this paper.

A CAR model is usually used for univariate spatial data, the data involving a single response variable. For multivariate spatial data which involve more than one response variable, the MCAR model proposed by [14] is commonly applied. An advantage of an MCAR model is that it can handle correlations between the response variables as well as the spatial correlations between areas. Reference [15] used MCAR for multivariate areal boundary analysis. They illustrated the methods using Minnesota county-level esophagus, larynx, and lung cancer data.

For this study, we choose rice and cassava yields to be forecasted because they are major crops of Thailand. Rice has played a vital role in Thailand's socio-economic development. It is the main export, and rice farming is a significant source of rural income. Thailand has the fifth-largest amount of land under rice cultivation in the world [16]. Rice is cultivated in about 8.93 million hectares of non-irrigated area, and the remaining 4.4 million hectares is cultivated in irrigated areas. About 40% of the total rice production is exported Cassava is one of the most important economic crops of Thailand. It can be used in an array of foods or as animal feed, ethanol, flour or starch, and is used in baking and cooking. Thailand is the fourth largest cassava producer in the world; however, it is the world largest exporter with export value of over THB 29 billion per year. Thailand's cassava planted area is 1.2 million hectares with a production yield of 26.9 million tons [1].

This study proposes an LMM with an MCAR model representing spatial effects, and a linear trend representing temporal effects, which is the extension of our previous model [12], for rice and cassava yields in 19 northeastern provinces of Thailand. Our previous model was an MCAR for spatial time series data without trend.

The proposed model, MCAR with trend, was compared with exponential smoothing with trend (Holt ES) which is a popular method for the trend data and also compared with MCAR model without trend from our previous study [12]. This paper is organized as follows. Section 2 briefly describes the methodology. The application is illustrated in Section 3. The results of the study are presented in Section 4. Lastly, the discussion and conclusions are presented in Sections 5 and 6, respectively.

2. Methodology

2.1 Linear mixed model (LMM) for time series data

A standard form of a linear mixed model is expressed as:

$$y_{it} = \boldsymbol{X}_{it}^T\boldsymbol{\beta} + \boldsymbol{Z}_{it}^T\boldsymbol{b}_i + \varepsilon_{it} \quad (1)$$

For $i = 1, \dots, m; t = 1, \dots, T$, where y_{it} is the $\boldsymbol{i}^{th}$ response at time t, $\boldsymbol{X}_{it}$ are the explanatory variables associated with the fixed effects, $\boldsymbol{\beta}$, $\boldsymbol{Z}_{it}$ correspond to the explanatory variables with random effects, $\boldsymbol{b}_i \sim MN(\boldsymbol{0}, \boldsymbol{D})$ where $\boldsymbol{D}$ is the positive definite matrix, and ε_{it} are the random errors which are normally independent and identically distributed (i.i.d.), $\varepsilon_{it} \sim N(0, \sigma^2)$.

2.2 Multivariate conditional auto regressive model (MCAR)

The MCAR model is described by [14] as follows. Let areal random effects corresponding to the two crop yields be $\boldsymbol{\varphi} = (\boldsymbol{\varphi}_1^T, \boldsymbol{\varphi}_2^T)$ where $\boldsymbol{\varphi}_1^T = (\emptyset_{11}, \dots, \emptyset_{m1})$, $\boldsymbol{\varphi}_2^T = (\emptyset_{12}, \dots, \emptyset_{m2})$, and m is the number of areal units. The bivariate spatial random effect $\boldsymbol{\varphi}$ is defined as the conditional distribution,

$$\begin{pmatrix} \emptyset_{i1} \\ \emptyset_{i2} \end{pmatrix} | \boldsymbol{\varphi}_{-(i1,i2)} \sim N\left(\begin{pmatrix} \overline{\emptyset}_{i1} \\ \overline{\emptyset}_{i2} \end{pmatrix}, (w_{i+}\boldsymbol{\Lambda})^{-1}\right) \quad (2)$$

where $\boldsymbol{\varphi}_{-(i1,i2)}$ stands for the collection of all $\emptyset_{il}$ except $\emptyset_{i1}$ and $\emptyset_{i2}$. Let $\overline{\emptyset}_{i1} = \sum_l \frac{w_{il}\emptyset_{l1}}{w_{i+}}$ and $\overline{\emptyset}_{i2} = \sum_l \frac{w_{il}\emptyset_{l2}}{w_{i+}}$, the averages of the random effects for area i's neighbors specific to variables 1 and 2, respectively. It can be seen that $\boldsymbol{\Lambda}$ serves as scaled conditional precision for $(\emptyset_{i1}, \emptyset_{i2})$, where w_{i+} is a scale parameter.

Since $\boldsymbol{\Lambda}$ is common for all areas $i = 1, \dots, m$, it controls the conditional precision for each pair of variables at the same site averaged over all areas. Letting $\sum = \boldsymbol{\Lambda}^{-1}$, $\frac{1}{w_+}\boldsymbol{\Sigma}$ is the conditional covariance matrix with $\rho_{12} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}\sigma_{22}}}$ as the conditional correlation between $\emptyset_{i1}$ and $\emptyset_{i2}$, $i = 1, \dots, m$. Under the MCAR, the multivariate joint distribution is

$$p(\boldsymbol{\varphi}) \propto exp\left(-\frac{1}{2}\boldsymbol{\varphi}^T[\boldsymbol{\Lambda} \otimes (\boldsymbol{D}_w - \boldsymbol{W})\boldsymbol{\varphi}]\right) \quad (3)$$

where $\boldsymbol{\Lambda}$ is 2x2 positive definite and $\otimes$ denotes the Kronecker product. $\boldsymbol{W} = (w_{ij})$ is a neighborhood matrix for areal units, which can be defined as

$$w_{ij} = \begin{cases} 1 \; if\ subregions\ i\ and\ j\ share\ a\ common \\ \qquad\qquad boundary, i \neq j \\ 0 \; otherwise \end{cases}$$

$\boldsymbol{D}_w = diag(w_{i+})$ is a diagonal matrix with (i, i) entry equal to $w_{i+} = \sum_j w_{ij}$.

2.3 Bayesian models

A Bayesian model usually consists of three stages of hierarchy. At the first stage, a linear model is set up given fixed and random effects; at the second stage, the distributions of fixed and random effects are specified given the variance components; finally, at the last stage, prior distributions are assigned to the variance components.

Reference [17] briefly described the basic elements of Bayesian inferences. Suppose that $\boldsymbol{y}$ is a vector of observations, $\boldsymbol{y} = (y_1, \dots, y_m)^T$, and $\boldsymbol{\theta}$ is a vector of parameters, $\boldsymbol{\theta} = (\theta_1, \dots, \theta_k)^T$. Let $f(\boldsymbol{y}|\boldsymbol{\theta})$ represent the conditional probability density function of $\boldsymbol{y}$ given $\boldsymbol{\theta}$, and $\pi(\boldsymbol{\theta})$ is a prior distribution for $\boldsymbol{\theta}$. Then, the posterior probability density function of $\boldsymbol{\theta}$ is given by

$$\pi(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{f(\boldsymbol{y}|\boldsymbol{\theta})}{f(\boldsymbol{y}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})d(\boldsymbol{\theta})} \tag{4}$$

or

$$\pi(\boldsymbol{\theta}|\boldsymbol{y}) \propto= f(\boldsymbol{y}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

A Bayesian point estimator for a univariate θ is often obtained as the posterior mean:

$$\begin{aligned} E(\theta|\boldsymbol{y}) &\propto \int \theta\pi(\theta|\boldsymbol{y})\, d\theta \\ &\propto \int \theta f(\boldsymbol{y}|\theta)\, \pi(\theta) d\theta \end{aligned} \tag{5}$$

However, maintaining and using this distribution often involves computing integrals which, for most non-trivial models, are intractable. Sampling algorithms based on Markov chain Monte Carlo (MCMC) techniques are one possible way to go about inference in such models. The underlying logic of MCMC sampling is that we can estimate any desired expectation by employing ergodic averages. That is, we can compute any statistic of a posterior distribution as long as we have N simulated samples from that distribution.

2.4 Markov chain Monte Carlo (MCMC)

An MCMC method is a general simulation method for sampling from the posterior distributions and computing posterior quantities of interest. The MCMC method samples successively from a target distribution. Each sample depends on the previous one, hence the notion of the Markov chain. A Markov chain is a sequence of random variables, $(\theta^1, \theta^2, \theta^3, \dots)$, for which the random variable θ^t depends on all previous θs only through its immediate predecessor θ^{t-1}. The Markov chain is applied to sampling as a mechanism that traverses randomly through a target distribution without having any memory of where it has been. Where it moves next is entirely dependent on where it is now. Monte Carlo is mainly used to approximate an expectation by using the Markov chain samples. In the simplest version

$$E\big(g(\theta)\big) = \int g(\theta)\, \pi(\theta) d\theta \cong \frac{1}{n}\sum_{t=1}^{n} g(\theta^t) \tag{6}$$

where $g(\cdot)$ is a function of interest and θ^t are samples from $\pi(\theta)$. This approximates the expected value of $g(\theta)$. Gibbs sampling is one of the MCMC techniques suitable to obtain samples from the posterior distribution. The idea in Gibbs sampling is to generate posterior samples by sweeping through each variable (or block of variables) to sample from its conditional distribution with the remaining variables fixed at their current values.

The Gibbs sampling [18-19] decomposes the joint posterior distribution into full conditional distributions for each parameter in the model and then samples from them. The sampler is efficient when the parameters are not highly dependent on each other and the full conditional distributions are easy to sample from. It does not require an instrumental proposal distribution as Metropolis methods do. However, while deriving the conditional distributions can be relatively easy, it is not always possible to find an efficient way to sample from these conditional distributions.

Suppose $\boldsymbol{\theta} = (\theta_1, \dots, \theta_k)^T$ is the parameter vector, $f(\boldsymbol{y}|\theta)$ is the likelihood, and $\pi(\boldsymbol{\theta})$ is the prior distribution. The full posterior conditional distribution of $p(\theta_i|\theta_j, i \neq j, \boldsymbol{y})$ is proportional to the joint posterior density; that is,

$$\pi(\theta_i|\theta_j, i \neq j, \boldsymbol{y}) \propto f(\boldsymbol{y}|\theta)\pi(\boldsymbol{\theta}) \tag{7}$$

For instance, the one-dimensional conditional distribution of θ_1 given $\theta_j = \theta_j^*, 2 \leq j \leq k$, is computed as the following:

$$\pi(\theta_1|\theta_j^*, 2 \leq j \leq k, \boldsymbol{y}) = f(\boldsymbol{y}|\boldsymbol{\theta} = (\theta_1, \theta_2^*, \dots, \theta_k^*)^T)\pi(\boldsymbol{\theta} = (\theta_1, \theta_2^*, \dots, \theta_k^*)^T) \tag{8}$$

The Gibbs sampling works as follows:

Step 1: Set $t = 0$, and choose an arbitrary initial value of $\boldsymbol{\theta}^0 = \theta_1^0, \dots, \theta_k^0$.

Step 2: Generate each component of $\boldsymbol{\theta}$ as follows:

- draw $\theta_1^{(t+1)}$ from $\pi(\theta_1|\theta_j^{(t)}, \dots, \theta_k^{(t)}, \boldsymbol{y})$
- draw $\theta_2^{(t+1)}$ from $\pi(\theta_2|\theta_1^{(t)}, \theta_3^{(t)} \dots, \theta_k^{(t)}, \boldsymbol{y})$
- . . .
- draw $\theta_k^{(t+1)}$ from $\pi(\theta_k|\theta_1^{(t+1)}, \theta_2^{(t+1)} \dots, \theta_{k-1}^{(t+1)}, \boldsymbol{y})$

Step 3: Set $t = t + 1$. If $t < T$, the number of desired samples, return to Step 2. Otherwise, stop.

3. Application

The rice and cassava yields (Unit: Tons) in 19 northeastern provinces of Thailand, extracted from the annual report of the Office of Agricultural Economics [1] from 2002 to 2011 (120 months), are used. The data are divided into 2 parts; the first 108 months are used for model fitting and the last 12 months are reserved for model validation. The proposed model which is a special case of LMM is applied to those data. It is expressed as follows.

Let z_{ikt} be the agricultural yield in province $i, i = 1, \dots, 19$, product type $k, k = 1$ for rice and $k = 2$ for cassava, and month $t, t = 1, \dots, 120$. We transform the data using the natural logarithmic function to make the data a more normal distribution [20].

$$\begin{aligned} y_{ikt} &= ln(z_{ikt} + 1) \\ y_{ikt} &= V_k + b_{kt} + \emptyset_{ik} + \beta * t + \varepsilon_{ikt} \\ y_{ikt} &= |V_k, \emptyset_{ik} \sim N(\mu_{ikt}, \sigma^2) \end{aligned} \tag{9}$$

where $\mu_{ikt} = V_k + b_{kt} + \emptyset_{ik} + \beta * t$ and $\varepsilon_{ikt} \sim N(0, \sigma^2)$. V_k are the product type random effects, b_{kt} are random effects of representing the baseline of product k and time t, $\emptyset_{ik}$ are the area-product type spatial effects, $\beta * t$ are the linear trends, and ε_{ikt} are province-product type-time random effects. The estimated μ_{ikt} are used for prediction.

3.1 Model estimation

Bayesian inference via Gibbs sampling MCMC in Open BUGS software [21] for parameter estimation is used.

For Bayesian setting, we assume priors for the parameters as follows.

$$\begin{aligned} &V_k \sim N(0, \sigma^2), \\ &\sigma_v^2 \sim InvGamma\ (0.005, 0.005) \\ &\begin{pmatrix} \emptyset_{i1} \\ \emptyset_{i2} \end{pmatrix} |\boldsymbol{\varphi}_{-(i1,i2)} \sim MCAR\ in\ (2) \\ &\sigma^2 \sim InvGamma\ (0.005, 0.005), \\ &b_{kt} \sim N\ (0, \sigma_b^2), \\ &\sigma_b^2 \sim InvGamma\ (0.005, 0.005) \end{aligned}$$

3.2 Model comparison

The proposed model is compared with the well-known exponential smoothing model with trend (Holt ES) and MCAR without trend in our previous model [12] using the mean absolute error (MAE) criterion,

$$MAE = \frac{\sum_{t=1}^{n}|e_t|}{n} = \frac{\sum_{t=1}^{n}|Y_t-\hat{Y}_t|}{n} \qquad (10)$$

where $\hat{Y}_t$ is the forecast value and Y_t is the actual observation at time t, and $e_t = Y_t - \hat{Y}_t$ is the forecast error at time t. The Gibbs sampling MCMC is run for 11,000 iterations, with burn-in of 1,000. We assess the MCMC convergence of all model parameters by visual analysis of history and Kernel density plots.

4. Results

For MCMC convergence diagnostics [22], visual analysis is used. The history plots for some estimated means are shown in Figs. 1-4 and the kernel density plots are shown in Figs. 5-8. The chains move around the parameter spaces and the kernel densities do not indicate multimodality or lumpiness. These indicate that each parameter is converged to a stationary density.

The performance of the proposed model compared to the Holt ES and the MCAR without trend, using the mean absolute error (MAE) criterion, is shown in Table 1 and Table 2. For rice yield, in the fitting part, the proposed model has a better performance in most provinces compared to the MCAR without trend model (13/19 = 68.42%) and in all provinces compared to the Holt ES (19/19 = 100%). In the validation part, the proposed model is superior to the MCAR without trend (19/19 = 100%) and the Holt ES (19/19 = 100%) in all provinces.

For the cassava yield, in the fitting part, the proposed model has a better performance in most provinces compared to the MCAR without trend (10/19 = 52.63%) and the Holt ES (12/19 = 63.16%). In the validation part, the proposed model is superior to the MCAR without trend (19/19 = 100%) and the Holt ES (19/19 = 100%) in all provinces.

Some of the actual and predicted values of rice and cassava yield are presented in Fig. 9-12 and Tables 3 and 4. It can be seen that the predicted values and the actual values have the same pattern. For the months with high product yield, the errors in the fitting part are quite large, however the errors in the validation part are quite small. For example, in Month 11, Loei province had a rice yield of 160,657 tons with an error of 117,178.01 tons; in Month 58, Roi Et province had a cassava yield of 135,703 tons with an error of 101,046.82 tons. In the validation part, in Month 119, Loei province had a rice yield of 167,147 tons with an error of 2,321.94 tons; in Month 117, Roi Et province had a cassava yield of 46,570 tons with an error of 1,264.97 tons.

From the results, the proposed model is more effective than the comparison models for both rice and cassava yields.

5. Discussion

The LMM with MCAR for spatial effects and linear trend for temporal effects is applied to spatio-temporal time series data. It takes into account the spatial correlations following the first law of geography stating that “Everything is related to everything else, but near things are more related than distant things” [23]. It also accounts for the temporal correlations within the product type, which usually occurs in time series data. The proposed model is quite complex, so the traditional method for parameter estimation, such as maximum likelihood, cannot be used. Therefore, a Bayesian method can be adopted to solve this problem. The proposed model is applied to the rice and cassava yields in Thailand. The benefit is that, in one model, it can predict multiple product yields in multiple provinces at the same time. Since the real data set consists of many zeros and extreme values, logarithmic transformation is applied in order to make the data more normally distributed. Even though the Holt ES can detect the trend, it cannot work with multiple products and multiple provinces in one model at the same time. Moreover, it cannot deal with spatial correlation. Compared to the Holt ES and our previous MCAR without trend, the proposed model has a better performance in most provinces in the fitting part and all provinces in the validation part.

The proposed model does not work very well in detecting extreme values. The reason is that there might be some outliers in the data. The limitation of this study is using secondary data, which causes problems of verification. For further study the proposed model can be extended to include outliers and seasonal components.

6. Conclusions

This study proposes an appropriate forecasting model for multivariate spatio-temporal time series data. The Bayesian approach using Gibbs sampling in MCMC for an LMM with linear trend for temporal effects and an MCAR for spatial effects is considered. The proposed model is applied to rice and cassava yields in 19 Northeastern provinces of Thailand from 2002 to 2011. Using the MAE criterion, the proposed model shows a better performance than the Holt ES and the MCAR without trend from our previous study in most provinces in the fitting part and all provinces in the validation part for both rice and cassava yields. The superiority of the proposed LMM with MCAR with trend is that, in one model, it can forecast several products in many provinces concurrently.

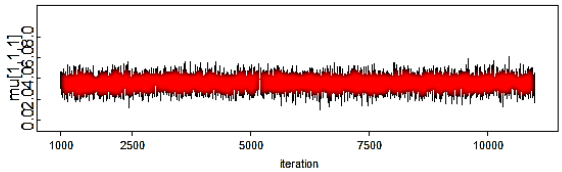

Fig. 1 History plot of the estimated mean for rice yield in January in Loei province

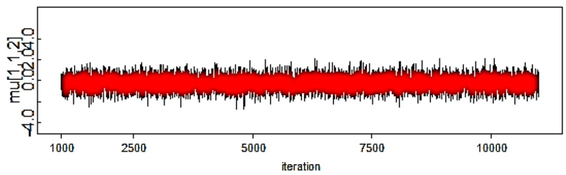

Fig. 2 History plot of the estimated mean for rice yield in February in Loei province

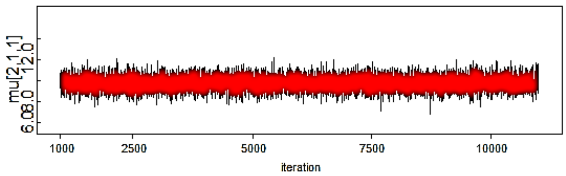

Fig. 3 History plot of the estimated mean for cassava yield in January in Loei province

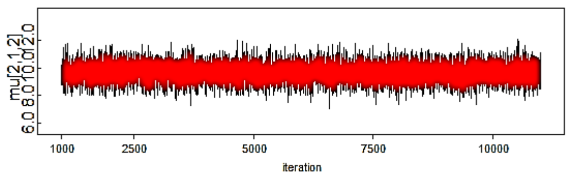

Fig. 4 History plot of the estimated mean for cassava yield in February in Loei province

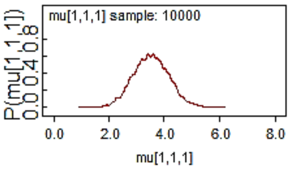

Fig. 5 Kernel density plot of the estimated mean for rice yield in January in Loei province

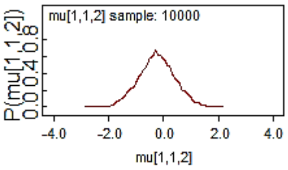

Fig. 6 Kernel density plot of the estimated mean for rice yield in February in Loei province

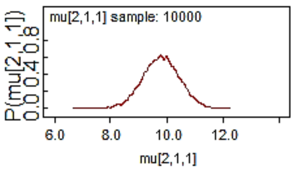

Fig. 7 Kernel density plot of the estimated mean for cassava yield in January in Loei province

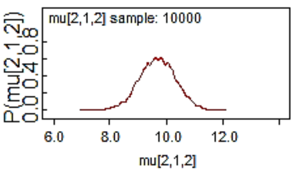

Fig. 8 Kernel density plot of the estimated mean for cassava yield in February in Loei province

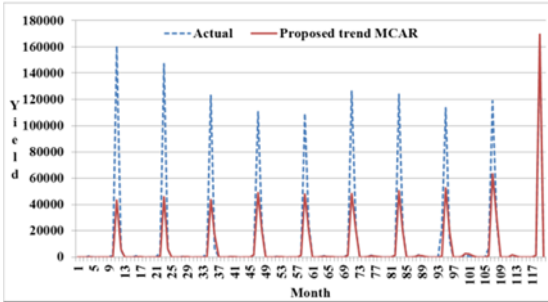

Fig. 9 Actual and predicted values of rice yield in Loei province

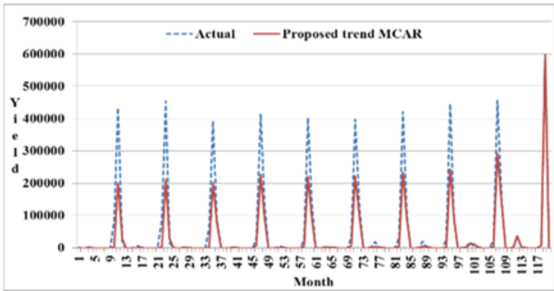

Fig. 10 Actual and predicted values of rice yield in Sakon Nakhon province

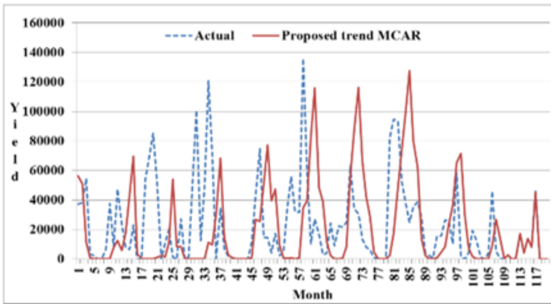

Fig. 11 Actual and predicted values of cassava yield in Roi Et province

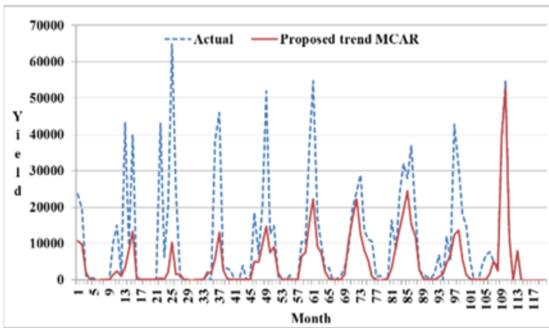

Fig. 12 Actual and predicted values of cassava yield in Nong Bua Lam Phu province

Table 1 Performance of the proposed trend MCAR, MCAR and Holt ES models for rice yield

Province	Model	MAE (Tons)	
		Fitting	Validation
Loei	Trend MCAR	**7,122.83**	**199.32**
	MCAR	43,760.09	68,576.67
	Holt ES	21,431.75	27,367.39
Nong Bua Lam Phu	Trend MCAR	**16,947.32**	**427.38**
	MCAR	32,741.48	68,987.50
	Holt ES	36,566.73	44,350.60
Udon Thani	Trend MCAR	**34,883.31**	**767.29**
	MCAR	44,974.35	68,970.00
	Holt ES	75,359.15	111,301.50
Nong Khai	Trend MCAR	**28,401.47**	**509.40**
	MCAR	34,566.30	68,716.67
	Holt ES	42,485.12	60,887.15
Sakon Nakhon	Trend MCAR	**22,182.55**	**246.15**
	MCAR	30,262.69	68,781.67
	Holt ES	74,836.55	100,763.84
Nakhon Phanom	Trend MCAR	**16,975.75**	**1,066.15**
	MCAR	34,867.41	68,503.33
	Holt ES	42,540.96	66,222.90
Mukdahan	Trend MCAR	**5,708.71**	**176.95**
	MCAR	46,269.07	68,607.50
	Holt ES	16,208.59	25,275.76
Yasothon	Trend MCAR	**15,689.88**	**704.62**
	MCAR	31,437.13	68,802.50
	Holt ES	43,974.76	64,928.48
Amnat Charoen	Trend MCAR	**15,548.68**	**824.31**
	MCAR	31,961.30	68,640.00
	Holt ES	40,232.09	55,360.44
Ubon Ratchathani	Trend MCAR	77,582.83	**2,253.67**
	MCAR	**51,923.06**	68,676.67
	Holt ES	119,794.94	188,814.72
Si Sa Ket	Trend MCAR	**32,734.60**	**2,748.53**
	MCAR	41,355.09	68,463.33
	Holt ES	113,493.36	179,994.35
Surin	Trend MCAR	55,081.40	**4,704.89**
	MCAR	**52,275.19**	68,817.50
	Holt ES	137,032.41	187,880.82
Buri Ram	Trend MCAR	52,132.21	**1,066.21**
	MCAR	**44,746.57**	68,914.17
	Holt ES	120,832.69	175,515.34
Maha Sarakham	Trend MCAR	**19,460.29**	**799.30**
	MCAR	23,225.65	69,092.50
	Holt ES	81,379.92	116,443.00
Roi Et	Trend MCAR	**29,770.46**	**1,204.96**
	MCAR	44,513.70	68,938.33
	Holt ES	121,254.18	165,547.40
Kalasin	Trend MCAR	49,998.18	**1,229.87**
	MCAR	**32,384.81**	68,933.33
	Holt ES	64,121.59	90,619.75
Khon Kaen	Trend MCAR	54,459.00	**2,037.55**
	MCAR	**37,123.80**	68,550.83
	Holt ES	91,091.59	126,403.82
Chaiyaphum	Trend MCAR	**26,453.99**	**270.03**
	MCAR	35,388.61	68,837.50
	Holt ES	52,521.49	68,769.46
Nakhon Ratchasima	Trend MCAR	60,777.03	**800.21**
	MCAR	**53,938.24**	68,470.00
	Holt ES	124,403.46	174,326.96

Table 2 Performance of the proposed trend MCAR, MCAR and Holt ES models for cassava yield

Province	Model	MAE (Tons)	
		Fitting	Validation
Loei	Trend MCAR	31,236.27	**961.80**
	MCAR	**30,714.54**	68,555.83
	Holt ES	40,759.85	51,643.56
Nong Bua Lam Phu	Trend MCAR	**6,715.40**	**201.03**
	MCAR	26,953.98	68,764.17
	Holt ES	10,273.92	11,012.26
Udon Thani	Trend MCAR	37,900.46	**451.44**
	MCAR	**28,730.65**	68,702.50
	Holt ES	31,380.85	67,017.49
Nong Khai	Trend MCAR	**10,698.73**	**129.29**
	MCAR	27,406.02	69,055.00
	Holt ES	13,787.09	9,857.80
Sakon Nakhon	Trend MCAR	**14,163.45**	**215.49**
	MCAR	26,374.17	68,630.83
	Holt ES	18,689.67	15,018.78
Nakhon Phanom	Trend MCAR	**2,851.31**	**22.88**
	MCAR	34,269.72	68,647.50
	Holt ES	3,849.55	3,785.27
Mukdahan	Trend MCAR	**18,699.48**	**237.36**
	MCAR	27,448.80	68,700.00
	Holt ES	24,197.74	31,597.77
Yasothon	Trend MCAR	**9,245.33**	**130.87**
	MCAR	26,151.85	68,680.00
	Holt ES	12,382.92	17,421.14
Amnat Charoen	Trend MCAR	**6,879.14**	**72.49**
	MCAR	27,258.70	68,700.00
	Holt ES	7,347.21	7,519.95
Ubon Ratchathani	Trend MCAR	**23,904.27**	**837.96**
	MCAR	28,071.20	68,671.67
	Holt ES	31,064.82	48,460.70
Si Sa Ket	Trend MCAR	**13,928.20**	**518.94**
	MCAR	25,591.30	68,810.83
	Holt ES	20,385.71	27,204.94
Surin	Trend MCAR	**7,744.78**	**287.55**
	MCAR	31,560.46	68,524.17
	Holt ES	9,082.64	9,596.62
Buri Ram	Trend MCAR	46,537.35	**591.19**
	MCAR	**32,408.61**	68,965.83
	Holt ES	50,476.74	50,387.92
Maha Sarakham	Trend MCAR	87,854.52	**323.95**
	MCAR	34,198.15	68,935.00
	Holt ES	**16,972.74**	19,502.47
Roi Et	Trend MCAR	27,475.63	**153.98**
	MCAR	35,771.02	68,890.83
	Holt ES	**20,639.98**	8,910.97
Kalasin	Trend MCAR	258,728.41	**967.51**
	MCAR	59,823.52	68,857.50
	Holt ES	**54,798.61**	48,533.87
Khon Kaen	Trend MCAR	187,020.45	**711.12**
	MCAR	38,935.28	68,656.67
	Holt ES	**35,977.21**	39,898.89
Chaiyaphum	Trend MCAR	273,015.43	**2,028.84**
	MCAR	**41,492.31**	68,618.33
	Holt ES	46,654.99	126,286.46
Nakhon Ratchasima	Trend MCAR	1,075,257.99	**10,252.06**
	MCAR	308,218.24	68,813.33
	Holt ES	**229,401.12**	415,007.79

Table 3 Actual values, predicted values, and absolute errors of rice in Loei and cassava in Roi Et in the fitting part from the proposed trend MCAR

	Rice in Loei Province (Tons)			Casava in Roi Et Province (Tons)		
Month	Actual	Predicted	Abs. Error	Actual	Predicted	Abs. Error
1	0	34.23	34.23	37,323	56,144.94	18,821.94
2	0	0.79	0.79	38,151	50,794.00	12,643.00
3	0	6.33	6.33	54,678	11,150.45	43,527.55
4	762	144.93	617.07	3,471	824.26	2,646.74
5	0	33.08	33.08	2,452	226.57	2,225.43
6	0	3.21	3.21	0	79.84	79.84
7	0	0.35	0.35	764	105.94	658.06
8	0	0.19	0.19	6,719	117.78	6,601.22
9	0	0.31	0.31	37,546	219.72	37,326.28
10	2,178	361.27	1,816.73	8,463	7,873.97	589.03
11	160,657	43,478.99	117,178.01	47,418	12,553.79	34,864.21
12	5,257	5,820.88	563.88	23,615	5,748.42	17,866.58
13	0	27.72	27.72	7,671	18,043.35	10,372.35
14	0	0.27	0.27	6,386	43,261.57	36,875.57
15	0	4.22	4.22	22,934	69,662.64	46,728.64
16	846	182.49	663.51	0	3,710.79	3,710.79
17	0	71.02	71.02	0	304.30	304.30
18	0	4.34	4.34	54,143	273.80	53,869.20
19	0	0.31	0.31	69,181	257.88	68,923.12
20	0	0.26	0.26	85,201	208.05	84,992.95
21	0	0.18	0.18	52,821	1,078.88	51,742.12
22	3,741	487.05	3,253.95	0	2,300.77	2,300.77
23	147,024	45,723.05	101,300.95	10,836	1,362.02	9,473.98
24	5,863	6,298.69	435.69	20,264	11,169.59	9,094.41
25	0	35.69	35.69	1,304	54,011.83	52,707.83
26	0	0.27	0.27	0	8,316.53	8,316.53
27	0	29.70	29.70	27,006	8,559.96	18,446.04
28	623	164.54	458.46	0	213.58	213.58
29	0	40.41	40.41	1,788	330.11	1,457.89
30	0	3.42	3.42	51,442	44.40	51,397.60
31	0	0.63	0.63	100,724	132.71	100,591.29
32	0	0.18	0.18	12,479	128.24	12,350.76
33	0	0.23	0.23	49,133	225.50	48,907.50
34	3,880	60.21	3,819.79	120,706	11,408.58	109,297.42
35	123,182	43,583.17	79,598.83	70,820	9,748.91	61,071.09
36	6,536	17,130.81	10,594.81	397	30,316.64	29,919.64
37	0	7.17	7.17	34,690	68,229.72	33,539.72
38	0	0.26	0.26	6,754	14,644.77	7,890.77
39	0	2.91	2.91	2,210	2,965.20	755.20
40	312	105.55	206.45	1,043	774.96	268.04
41	49	141.04	92.04	0	131.50	131.50
42	0	1.39	1.39	0	93.41	93.41
43	0	0.27	0.27	0	202.96	202.96
44	0	0.18	0.18	0	99.38	99.38
45	0	0.32	0.32	11,696	837.22	10,858.78
46	1,594	1,574.26	19.74	42,603	26,763.74	15,839.26
47	110,682	49,149.68	61,532.32	74,760	25,669.32	49,090.68
48	23,198	20,025.55	3,172.45	14,596	51,050.54	36,454.54
49	0	0.42	0.42	14,399	77,264.01	62,865.01
50	0	0.53	0.53	4,349	39,534.58	35,185.58
51	0	32.17	32.17	17,199	47,508.54	30,309.54
52	750	332.79	417.21	2,362	9,506.08	7,144.08
53	21	233.47	212.47	4,626	932.31	3,693.69
54	0	17.29	17.29	34,827	609.83	34,217.17
55	0	0.83	0.83	56,046	893.32	55,152.68
56	0	0.18	0.18	32,224	216.70	32,007.30
57	0	0.32	0.32	31,496	912.92	30,583.08
58	755	1,128.83	373.83	135,703	34,656.18	101,046.82
59	109,244	47,894.14	61,349.86	59,337	39,584.99	19,752.01
60	24,920	20,379.12	4,540.88	10,290	82,538.90	72,248.90
61	0	0.61	0.61	27,105	116,019.13	88,914.13
62	0	1.25	1.25	16,636	48,309.54	31,673.54
63	59	84.91	25.91	3,191	38,886.52	35,695.52
64	994	610.66	383.34	3,944	11,721.78	7,777.78
65	84	385.93	301.93	24,201	2,502.86	21,698.14
66	0	48.72	48.72	8,963	107.44	8,855.56
67	0	0.68	0.68	22,372	316.91	22,055.09
68	0	0.18	0.18	21,870	444.93	21,425.07
69	0	0.32	0.32	24,918	8,521.83	16,396.17
70	1,850	1,253.23	596.77	63,370	56,132.02	7,237.98
71	126,560	48,179.06	78,380.94	34,243	87,157.02	52,914.02
72	20,818	20,193.81	624.19	30,532	116,141.36	85,609.36
73	0	0.57	0.57	13,577	66,383.65	52,806.65
74	0	0.78	0.78	8,260	42,416.85	34,156.85
75	113	158.06	45.06	6,187	28,691.82	22,504.82
76	1,339	851.95	487.05	1,237	5,010.31	3,773.31
77	117	626.84	509.84	0	118.16	118.16
78	0	243.23	243.23	0	47.23	47.23
79	0	2.24	2.24	0	149.31	149.31
80	0	0.18	0.18	82,365	2,354.13	80,010.87
81	0	0.32	0.32	94,638	17,202.76	77,435.24
82	1,173	1,126.54	46.46	93,387	48,354.67	45,032.33
83	124,043	50,513.28	73,529.72	52,208	74,125.15	21,917.15
84	21,442	19,910.32	1,531.68	38,132	99,452.49	61,320.49
85	0	0.56	0.56	25,036	127,530.40	102,494.40
86	0	0.80	0.80	35,331	80,165.16	44,834.16
87	39	140.17	101.17	38,832	61,872.16	23,040.16
88	1,682	1,230.69	451.31	28,398	13,906.47	14,491.53
89	109	999.73	890.73	4,377	2,285.06	2,091.94
90	0	169.86	169.86	2,451	262.08	2,188.92
91	0	1.88	1.88	910	311.23	598.77
92	0	0.18	0.18	15,337	695.82	14,641.18
93	0	0.18	0.18	15,757	4,689.60	11,067.40
94	22,180	615.84	21,564.16	26,433	8,378.35	18,054.65
95	113,527	52,617.54	60,909.46	25,648	24,047.10	1,600.90
96	12,774	19,481.98	6,707.98	10,259	36,315.80	26,056.80
97	0	1.14	1.14	58,429	65,041.76	6,612.76
98	0	1.45	1.45	1,844	71,484.40	69,640.40
99	83	349.26	266.26	0	30,031.44	30,031.44
100	1,715	2,590.62	875.62	4,168	7,391.84	3,223.84
101	438	2,473.77	2,035.77	19,236	1,877.30	17,358.70
102	0	804.32	804.32	12,824	169.72	12,654.28
103	0	3.84	3.84	1,363	229.79	1,133.21
104	0	0.18	0.18	0	268.81	268.81
105	0	0.18	0.18	3,847	173.00	3,674.00
106	6,656	861.28	5,794.72	46,071	8,999.83	37,071.17
107	118,841	63,186.77	55,654.23	5,326	26,971.70	21,645.70
108	25,087	28,774.50	3,687.50	171	13,193.70	13,022.70
MAE			**7,122.83**			**27,475.63**

Table 4 Actual values, predicted values, and absolute errors of rice in Loei and cassava in Roi Et in the validation part from the proposed trend MCAR

	Rice in Loei Province (Tons)			Casava in Roi Et Province (Tons)		
Month	Actual	Predicted	Abs. Error	Actual	Predicted	Abs. Error
109	0	0.98	0.98	482	472.67	9.33
110	0	1.00	1.00	2,782	2,711.28	70.72
111	31	30.35	0.65	0	1.00	1.00
112	1,517	1,484.76	32.24	370	364.87	5.13
113	408	404.14	3.86	16,965	17,205.88	240.88
114	0	1.01	1.01	4,271	4,255.31	15.69
115	0	0.95	0.95	13,758	13,832.59	74.59
116	0	1.03	1.03	8,290	8,127.56	162.44
117	0	1.02	1.02	46,570	45,305.03	1,264.97
118	1,313	1,295.64	17.36	0	1.01	1.01
119	167,147	169,468.94	2,321.94	0	1.01	1.01
120	460	450.15	9.85	0	1.02	1.02
MAE			**199.32**			**153.98**

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