Space-Time Codes Technology

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ABSTRACT

Space-time codes technology is a channel coding for wireless digital communications, where multiple antennas are employed. It improves the capacity of the transmission as well as reducing errors. Also, this technology does not require the expansion of bandwidth or time slots. In order to achieve the highest efficiency, we have to first investigate the maximum efficiency that can be achieved. Then, the code design criteria for obtaining the maximum efficiency have to be derived. Last, the code design approaches have to be proposed. The article discusses those procedures.

Keyword: Space-time codes, space-frequency codes, channel coding, diversity

1. Introduction

The next-generation wireless systems are required to provide higher bit rates in the order of Mbits/s for applications in outdoor environments with high user mobility and in the order of Gbits/s for high-speed indoor services, supporting not only high quality voice service but also various emerging broadband applications such as real-time multimedia services and video conferencing, covering a large service area. To meet this requirement it is important to develop new wireless communication techniques that offer a high spectral efficiency better quality and more reliable transmission under power consumption constraint in diverse environments: macro, micro and picocellular; urban, suburban and rural.

The fundamental phenomenon which makes reliable wireless transmission difficult is time-varying multipath fading. It is this phenomenon which makes wireless transmission a great challenge when compared to wired-line such as fiber or coaxial cable transmissions. Unlike the additive white Gaussian noise (AWGN) channel, it is not possible to improve the effective bit error rate performance of digital communication over fading channel by increasing the transmit power. For example, in AWGN channel reducing the bit error rate from 10^{-2} to 10^{-3} may require only 1 or 2-dB higher signal-to-noise ratio (SNR). To achieve the same improvement in multipath fading channel, however, up to 10 dB increasing in SNR is required [1]. In addition, employing traditional channel coding alone cannot mitigate the effect of fading efficiently as much as achieved in AWGN channel.

For loss in SNR due to fading, the mitigation technique called for is to improve the received SNR (or reduce the required SNR). It is well-known that introducing some form of signal diversity is one of the efficient ways to accomplish this. Diversity techniques are based on the notion that errors occur in reception when the channel attenuation is large, i.e., when the channel is in a deep fade. If we can supply to the receiver several replicas of the same

information signal transmitted over independently fading channels, the probability that all the signal components will fade simultaneously is reduced considerably [2]. There are several ways in which we can provide the receiver with independently fading replicas of the same information-bearing signal. Common ways of diversity are time diversity (due to Doppler spread) and frequency diversity (due to delay spread). For decades, the use of spatial (or antenna) diversity has become increasingly popular [3]. Spatial diversity is particularly attractive since it can be provided without loss in spectral efficiency.

Traditionally, the approach to obtain spatial diversity is by using multiple antennas on the receive side associated with combining techniques, known as receive diversity, which is a well-studied subject [4]. Receive diversity method requires that a number of transmission paths be available, all carrying the same message but having independent fading statistics. Proper combination of the signals from the transmission paths result in reduced severity of fading and improved reliability of transmission. For example, the copies of received signals are combined in many ways: equal gain combining (EGC), orthogonality restoring combining (ORC), maximum ratio combining (MRC), and minimum mean square error combining (MMSEC) [5]. Driven by mobile wireless applications, however, where it is difficult to deploy multiple antennas in the handset, transmit diversity or equivalently the use of multiple antennas on the transmit side has become an active area of research [6]-[9]. Space-time coding evolved as one of the most promising transmit diversity techniques [10]-[13]. Space-time codes (STC) provide spatial diversity together with code gain without sacrificing the transmission rate or loads of processing. In other words, STC is the combination of transmit diversity in the form of multiple antennas coupled with error correction coding.

It is interesting to note that while the origins of receive diversity for combating multipath dated back to the classic work of Brennan [14], the notion of STC is, relatively speaking, quite recent. Perhaps the earliest traces of such idea appear in the work of Wittneben [15] and Seshadri and Winters [6], who suggested various delay transmit antenna diversity techniques for application in base stations. These works proclaim that sending the well-designed signals from more than one transmitter simultaneously can provide diversity with some processing at the receiver. However, these works sacrifice the effective gain in receive SNR by a factor equal to the number of antennas and the scheme is presented by only giving some examples of designing the signals. They do not develop an analytical basis or general rule of designing the signals. In any cases, they are the origin of the STC.

Later, STC became understood among researchers in the field. Since then, many new results were contributed at a fast pace. The contributions can be categorized into three types: a) establishing new code design criteria for different radio environments or constraints, b) proposing code designs with superior performance, and c) examining the properties of STC in different radio environments. The first objective of this article is to review these contributions. The second objective is to analyze them by commenting the results based on our own intensive investigations, and the third objective is to synthesize by suggesting the promising future work.

2. Space-Time Codes

The first two works that enlighten the principle of STC are established by Guey et. al. [10], and Tarokh et. al. [11]. In fact, these two works were done in parallel. The name "Space-Time Codes" (STC), which has been used ubiquitously since then, was given by the work of Tarokh et. al.

Furthermore, another pioneer work, proposed by Alamouti, introduces a special case of block codes that can be counted as a new class of space-time codes [12], which was named later as "Space-Time Block Codes" (STBC). The STBC is a candidate in wireless LAN's standard [16] because of its simplicity and giving powerful performance. Since Alamouti did not generalize the STBC to any number of transmitter antennas, many researchers tried to extend or generalize the class of STBCs. For less than a year, Tarokh et. al. was able to elaborate the paradigm of STBC [13]. Nevertheless, to consider the optimal design of STC, the available tool is the criteria; whereas, to the best of our knowledge, the systematic means to design the optimal STC has not been provided in the literature so far, thereby leaving the problem of code design to the generations of coding researchers that followed. Furthermore, the research on the interactions and combinations of the STC technology with other techniques or numerous other topics has been being pursued.

Yet, the aforementioned and most works about STC assume the knowledge of channel state information (CSI) at the receiver side, which is usually accepted in wireless communication using coherent detection. Consequently, designing the STC for a system having the knowledge of CSI at the transmitter side is the challenging issue. Theoretically, the most effective technique to mitigate multipath fading in a wireless channel is transmitter power control [12]. If channel conditions as experienced by the receiver on one side of the link are known at the transmitter on the other side, the transmitter can predistort the signal in order to overcome the effect of the channel at the receiver. Pragmatically, the transmitter does not have any knowledge of the channel experienced by the receiver except in systems where the uplink (mobile to base) and downlink (base to mobile) transmissions are carried over the same frequency. Hence, the channel information has to be fed back from the receiver to the transmitter, which results in throughput degradation and added complexity to both the transmitter and the receiver.

From the advantage and disadvantage of having CSI at the transmitter side explained above, it is interesting to exploit the benefit of providing CSI to the transmitter in STC scheme, but the trade-off from the throughput degradation and complexity should not offset the obtained improvement.

3. In the View of Information Theory

The outlook of the Shannon Theorem about channel capacity (its second part) [59] has been changed when there are multi-antenna. From the original,

$$C = W \log_2 \left(1 + \frac{E_s}{N_0} \right) \quad (b/s), \qquad (1)$$

or equivalently,

$$\frac{C}{W} = \log_2 \left(1 + \frac{E_s}{N_0} \right) \quad (b/s/Hz), \qquad (2)$$

where C is the capacity, W is the available bandwidth, and E_s/N_0 is energy-per-symbol -to-noise ratio in linear scale. The meaning is, arbitrary low error rate can be achieved at the E_s/N_0 which is enough for the given transmission rate. For instance, in order to send data at rate 1 b/s/Hz, receiver will need $E_s/N_0 = 0$ in decibel. Researchers will try effortlessly to find the codes which suppresses error rate to 10^{-5} at E_s/N_0 near 0 dB.

When there are multi-antenna, the theorem will be extended to be

$$\frac{C}{W} = \log_2 \left(\det \left(I + \frac{E_s}{N_0} HH^H \right) \right), \qquad (3)$$
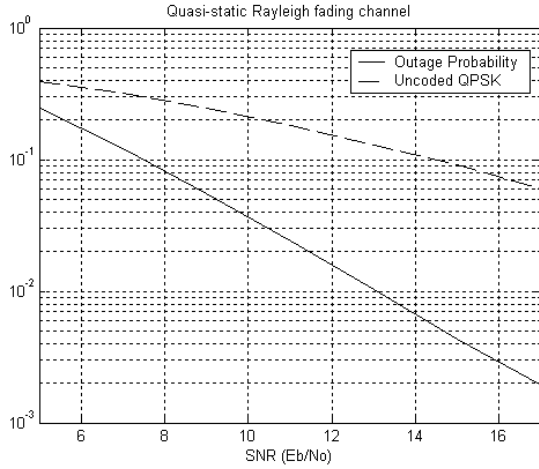
Figure 1 The outage probability of 2-Tx 1-Rx system at 2 b/s/Hz

where H is a matrix which the $\left(i^{th}, j^{th}\right)$ element is the path gain from transmit antenna i to receive antenna j, I is an identity matrix, and the unary operator ^{H} stands for Hermitian operator. Obviously, the capacity can be increased without sacrifice of bandwidth by deploying multi-antenna, depending on the independence among path gains. If all path gains are completely independent, Telatar showed that there are $M_T M_R$ levels of diversity available (the maximum diversity gain) and there are $\min\left(M_T, M_R\right)$ independent parallel channels that can be established (the maximum multiplexing gain), where M_T is the number of transmit antenna and M_R is the number of receive antenna [60]. These bounds were found independently by Foschini and Gans [61], which Foschini himself realized the maximum multiplexing gain in [62]. On the other hand, STC pursuits the maximum diversity gain. The optimal point of diversity and multiplexing simultaneously is investigated in [63]. In any case, since wireless communication channel has fading, the STC scheme approach is attractive for obtaining diversity together with code gain for mitigating

fading and combating noise without sacrifice of bandwidth - happening in using time diversity or frequency diversity.

The changed outlook in the Shannon's theorem is to use outage probability instead of the ergodic capacity. The outage probability is described by the following example. Let the elements of H are independent circular complex Gaussian random variable with unit variance representing 2-Tx 1-Rx system, and fix the desired capacity to 2 b/s/Hz. After sampling enough number of H, we use (3) to calculate the capacity of each sample of H for several values of SNR. If the obtained capacity is lower than 2 b/s/Hz, the outage happens. The rate of outage happening is plotted along those SNRs as shown in Figure 1. The outage probability is the bound of STC. In other words, researchers will try effortlessly to find the codes which frame error rate (FER) near the bound. The gain of STC is divided into diversity gain which decreases the slope of FER graph and coding gain which shifts the FER graph horizontally to the left.

4. Design Criteria

Assume the channel is the quasi-static Rayleigh fading channel. Let the sent codeword be c with length l. The received signal r is

$$r = cH + n, \tag{4}$$

where n is an AWGN matrix.

The pair wise error probability (PWEP) that codeword c can be decoded in favor of codeword e is

$$P\left(c \rightarrow e | H\right) = Q\left(\sqrt{\frac{E_s}{2N_0} d^2\left(c, e | H\right)}\right), \tag{5}$$

where

$$d^2\left(c, e | H\right) = \sum_{j=1}^{M_T} \sum_{t=1}^{l} \left| \sum_{i=1}^{M_R} h_{i,j}(t)\left(c_t^i - e_t^i\right) \right|^2. \tag{6}$$

Using the Chernoff bound $Q(x) \leq \exp(-x^2/2)$ to (5), we get

$$P(c \rightarrow e|H) \leq \exp\left(-d^2(c,e|H)\frac{E_s}{4N_0}\right). \quad (7)$$

With a knowledge about linear algebra, (7) can be manipulated in order to be able to statistically marginalized H. The result is

$$P(c \rightarrow e) \leq \left(\prod_{i=1}^{r}\lambda_i\right)^{-M_R}\left(\frac{E_s}{4N_0}\right)^{-rM_R}, \quad (8)$$

where r is the rank of A in (9), and λ is the eigenvalue of D, where

$$D = (c-e)(c-e)^H. \quad (9)$$

From (8), the slope of FER graph is proportional to rM_R, and the offset of the graph is proportional to $\left(\prod_{i=1}^{r}\lambda_i\right)^{M_R}$. After the M_R is fixed by the system, the rank and the product of eigenvalues could be maximized by designing the codes with the following criteria.

The criteria to accomplish the maximization are the rank criterion and the determinant criterion:-

4.1 The Rank Criterion

In order to achieve the maximum diversity $M_T M_R$, the matrix D has to be full rank for any codewords c and e. If D has minimum rank r over the set of two tuples of distinct codewords, then a diversity of rM_R is achieved.

4.2 The Determinant Criterion

Suppose that a diversity benefit of rM_R is our target. The minimum of rth roots of the sum of determinants of all $r \times r$ principal cofactors of D taken over all pairs of distinct codewords c and e and corresponds to the coding advantage, where is the rank of D. Special attention in the design must be paid to this quantity for any codewords c and e. The design target is making

this sum as large as possible. If a diversity of $M_T M_R$ is the design target, then the minimum of the determinant of D taken over all pairs of distinct codewords c and e must be maximized.

In the rapid Rayleigh fading channel, the received signal r_t, now depending on time instance and being vector, is

$$\vec{r}_t = \vec{c}_t H_t + \vec{n}_t, \quad (10)$$

and the PWEP is

$$P(c \rightarrow e) \leq \prod_{t \in v(c,e)}\left(|c_t - e_t|^2\frac{E_s}{4N_0}\right)^{-M_R}, \quad (11)$$

where $v(c,e)$ is the set which members are the time instances such that $c_t \neq e_t$. From (11), the criteria to accomplish the maximization are the distance criterion and the product criterion:-

4.3 The Distance Criterion

Let v be the cardinality of $v(c,e)$. In order to achieve the diversity vM_R in a rapid fading environment, for any two codewords c and e the sequence of $\vec{c}_t$ and $\vec{e}_t$ must be different at least for v time instances.

4.4 The Product Criterion

Let $v(c,e)$ denote the set of time instances $1 \leq t \leq l$ such that $\vec{c}_t \neq \vec{e}_t$. Then to achieve the most coding advantage in rapid fading environment, the minimum of the products $\prod_{t \in v(c,e)}|\vec{c}_t - \vec{e}_t|^2$ taken over distinct codewords c and e must be maximized.

The following is a simple example of STTC given in [11]. Let the system has 2-Tx (the number of Rx is not necessary to be determined), and the transmission rate is 2 b/s/Hz. Without exhaustive searching, a pragmatic approach to meet the rank criterion is to put the zero at the opposite corners of the matrix $c-e$ as shown in (12) to all distinct codewords c and e.

$$c - e = \begin{bmatrix} 0 & c_{1,2} - e_{1,2} \\ c_{2,1} - e_{2,1} & \vdots \\ \vdots & c_{l-1,2} - e_{l-1,2} \\ c_{l,1} - e_{l,1} & 0 \end{bmatrix} \quad (12)$$
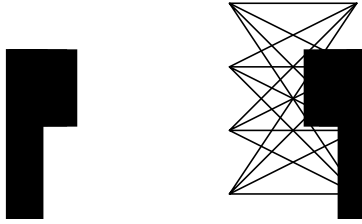

Figure 2 QPSK 4-state STTC of 2-Tx system at 2 b/s/Hz

If we would like to design full-rate 4-state STTC, the codes will be one shown in Figure 2. The diversity gain is maximized to 2 (throughout this proposal, the number of Rx is assumed to be 1) with the coding gain of 4.

The optimal decoding is the Viterbi algorithm. The measure using in decoding will be extended to the case of multi-antenna, as shown in (13).

$$\hat{c} = \arg\min_c \sum_{j=1}^{M_T} \left| \vec{r}_{r,j} - \sum_{i=1}^{M_R} h_{i,j} \vec{c}_{t,i} \right|^2 \quad (13)$$

5. Lines of Research

5.1 Space-Time Code

In addition to the discovery of the space-time code principle in [11], Tarokh also invented a class of STC from the founded principle which is named Space-Time Trellis-Code (STTC), while Guey also gave the rule to design the pilot signals for estimation of CSI at the receiver. A special case of STBC, i.e. two-antenna transmitter (2-Tx), was introduced independently of the principle of STC [12]. At that time, many researchers tried to find the STBC for other number of antennas until Tarokh and Jafarkhani generalized and elaborated the paradigm of STBC [13]. They proved that

STBC exist only in certain number of antennas and describe the codes in those possible cases. In fact, the contribution of this work is the indication that the STBC design is the orthogonal design. Even though everyone already knew the limitation of designing STBC, the research about STBC did not completely reach the dead end yet. Jafarkhani himself proposed the irregular STBC that permit the quasi-orthogonal design with the trade-off that only portion of diversity is obtained and modification at the decoder is needed [17].

Since the first set of STTC was exemplified in [11], there were considerable efforts to find the better set of STTC. Some of those works proposed code constructions while some works claimed their better codes searching results e.g. [18] and [19]. However, only marginal or moderate gains were archived. The dead end of STTC was broken through by the idea of merging STBC with Trellis-Coded Modulation (TCM, see [20], [21], and [22]) in order to be a new class of STTC. The traditional merging was done by serial concatenation of them, e.g. [23] and [24], which are rate-lossy. To make the full-rate code, the way of merging is changed to be embedding STBC into each branch of STTC's trellis structure as an alphabet, and the number of emanating branches from each state is maximized to provide full-rate coding. However, there are not enough STBC alphabets to fill all branches, making full-rate STBC-TCM unreachable. The identical solution of this problem was found independently by Siwamogsatham [25]-[28] and Jafarkhani [29]. They expand the number of available STBC alphabets by multiplying them with transformation matrix(s) and keep each set of alphabets from merging in the trellis structure. This solution is named "Super-Orthogonal Space-Time Trellis

Codes" (SOSTTC) by Jafarkhani, and "Expanded STBC-MTCM Construction" by Siwamogsatham. In fact, Siwamogsatham's works also give a point that the complexity of decoding can be reduced with the orthogonality of STBC, which enable space-time decoding to apply the pruning methodology used regularly in decoding MTCM codes [30]. The performance of SOSTTC is superior to all proposed STTC codes at that time. It should be noted that, the original STTC cannot contain parallel transitions in trellis structure; on the other hand, SOSTTC always has parallel transitions in trellis structure. Hence, only STBC-MTCM form does exist, and STBC-TCM form cannot be constructed.

5.2 Space-Frequency Code

Employing STC to OFDM system was investigated by Bolcskei and Paulraj in [3]. This work proclaimed that the existing STC may be applied directly to OFDM system by mapping time domain to frequency domain but the frequency diversity, which equals to the number of delay taps of ISI, will not be exploited. In order to exploit the frequency diversity, which can be thought as an extra diversity, and spatial diversity altogether, the new criteria are needed to be derived. They derived the new criteria that can be used to design the STC that exploit the extra diversity, and renamed the STC using in OFDM system to be Space-Frequency Codes (SFC). They exemplified later an SFC searched by their criteria in [31]. However, this example makes a huge loss in rate which is lower than the maximum rate. From Corollary 3.3.1 of [11], exploitation of extra diversity affects the maximum rate. Consequently, the obtained rate is much lower than the maximum rate. Su et. al. proposed a simple construction of SFC by only stacking the conventional STC [32] which provides the maximum rate, and also derived new criteria for designing SFC in [33]. The different treatment to ISI was explored by El Gamal et. al. in [34].

5.3 Specific Modulations

In [35], Nakamura and Torii considered the performance of Ternary Phase Shift Keying modulation (TPSK), and design a new class of convolutional codes exclusively for TPSK that do not need mapping from binary data symbol to ternary data symbol. The merit of TPSK is the less zero-crossing. From this point of view, it outperforms the standardized $\pi/4$-QPSK modulation used in Digital Cellular Systems (DCS) which is the 2nd generation cellular system using in North-America region. Koike and Yoshida searched the STTC for TPSK modulation in [36] and [37]. While $\pi/4$-shifted differential QPSK STBC was constructed in [38], TPSK STBC was proposed in [39].

In GSM standard, zero-crossing problem is mitigated by using continuous phase modulation (CPM) which is GMSK modulation. The traditional criteria used to design STC cannot be applied directly in this case due to its nonlinearity. The more appropriate criteria were provided in [50].

5.4 Combining with Turbo Codes

It is natural to combine the highly successful of Turbo Codes scheme in AWGN channel with the powerful of STC scheme in fading channel. Haimovich's group tried to combine these two schemes by strictly keeping the structure of Turbo-TCM (see [39] and [40]) that comprises the systematic stream and the punctured parity stream, which is built from two systematic recursive STTC [41]-[44]. Fitz's group used to innovate the binary rank criterion which is the new criterion that enables searching the STC in

algebraic framework [45]. This work transforms space-time codes design to algebraic framework design. According to the earlier innovation, Fitz's group applied binary rank criterion, a.k.a. $\sum_0$-rank criterion, to search the systematic recursive convolutional encoder as the constituent encoder of combining scheme [46] and [47]. Its structure is different from one of Haimovich's group. Vucetic's group kept the block diagram of Turbo-TCM but the constituent encoder was changed to be non-systematic recursive STTC encoder [48]. In [49], SOSTTC is used as the constituent encoder of Turbo-TCM which provides a significant improvement over conventional SOSTTC. In addition, the modification to the trellis structure of SOSTTC is necessary.

5.5 Combined Techniques

In [51], co-channel interference, which degrades the performance of STC, are suppressed by combing STC with the beam forming technique. In [52], codeword is generated by spanning of usual alphabets with basis matrices, of which elements can be any complex number. It is named "Linear Dispersive Codes" (LDC). The work in [53] gives derivation of pair wise error probability (PEP) of STC when the correlations among antennas, elaborated in [54]-[56], are present, and also find the indicator of robustness of STC to those correlations in case that they are present at the transmitter side. The SFC with adaptive bit allocation (bit-loaded) is constructed in [57]. Besides, STC expands its involvement to design of protocol, e.g. relay channel which provides cooperative diversity. Cooperative diversity is a transmission technique, where multiple terminals pool their resources to form a virtual antenna array that realizes spatial diversity gain in a distributed fashion [58].

6. Discussion

SOSTTC is the recommended in the case of frequency-flat fading, because it provides high coding gain and the decoding complexity can be reduced without performance trade-off. The main obstacle to maximize the coding gain is that computing the full search at the large number of states is not practical, because the full search is a combinatorial problem.

In the case of frequency-selective fading, trellis codes are expected to provide coding gain higher than that of block codes, but the number of states has to be high enough to obtain full diversity. Hence, the receiver suffers the high decoding complexity, and the block codes might be preferable in practice.

Since the proposed code design in the literature is quite close to the bound, the promising future work is to build a new class of space-time codes under specific constraints or limitations. For example, when the radio terminals move very fast, the wireless transmissions experiences the high Doppler spread, which does not obey the basic assumption that the channel is approximately static during the codeword interval. Hence, a new class of space-time codes should be established to maximize the diversity gain and coding gain.

7. Conclusion

The development of space-time codes has progressed through several lines of research. The space-time codes technology is sufficiently transparent for combining with various techniques. Even though the efficiency of the space-time codes in the literature came close to the limit, there are some new issues to explore, e.g., reducing complexity without sacrificing efficiency.

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