ชุดกิจกรรมวิทยาศาสตร์ทางเลือกสำหรับหาความหนาแน่นของของเหลวด้วยหลักการของอาร์คิมิดีส

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บทคัดย่อ

บทความนี้นำเสนอการพัฒนาชุดอุปกรณ์ทดลองอย่างง่าย สำหรับการคำนวณค่าความหนาแน่นของของเหลวจากมวลและปริมาตรของวัตถุที่ลอยอยู่ในของเหลว โดยอาศัยหลักการของอาร์คิมิดีส ชุดทดลองประกอบด้วยขวดทรงกระบอกจำนวน 5 ชิ้น ที่มีปริมาตรเท่ากันบรรจุทรายที่มีมวลแตกต่างกัน และมีความหนาแน่นน้อยกว่าของเหลวที่ต้องการระบุความหนาแน่น โดยการนำวัตถุนี้ไปลอยในของเหลว และบันทึกความสูงของวัตถุส่วนที่ลอยเหนือของเหลว จากนั้นเขียนกราฟความสัมพันธ์ระหว่างความสูงของวัตถุที่อยู่เหนือของเหลวในแกน y และมวลของวัตถุที่ลอยอยู่ในน้ำในแกน x ตามลำดับ ค่าความหนาแน่นของของเหลวและปริมาตรของวัตถุสามารถคำนวณได้จากค่าความชันและค่าจุดตัดบนแกน y ชุดอุปกรณ์ได้ถูกนำไปทดลองสอนนักเรียนชั้นมัธยมศึกษาปีที่ 6 ที่ต้องปรับพื้นฐานก่อนเข้าศึกษาต่อในระดับปริญญาตรีชั้นปีที่ 1 จำนวน 9 คน พบว่า คะแนนหลังเรียนมีค่าเพิ่มขึ้นภายหลังจากการใช้ชุดการทดลอง ซึ่งแสดงให้เห็นว่านักเรียนมีความเข้าใจเรื่องแรงลอยตัวเพิ่มขึ้น คาดว่าชุดกิจกรรมวิทยาศาสตร์อย่างง่ายชุดนี้จะสามารถในไปประยุกต์ใช้ในการจัดการเรียนการสอนเรื่องการลอยตัว และสมการเชิงเส้นสำหรับนักเรียนในระดับมัธยมศึกษาได้

คำสำคัญ: ความหนาแน่น แรงลอยตัว อาร์คิมิดีส ชุดกิจกรรมวิทยาศาสตร์

An Alternative Science Kit for Finding the Liquid Density Based on Archimedes' Principle

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Abstract

A simple experimental setup for finding the liquid density with known mass and volume of floating objects using Archimedes' was developed as a teaching tool for Archimedes' Principle. The instrumentation consists of a set of 5 similar-volume cylindrical bottles filled with sand with different masses whose overall densities are less than the liquid whose density needed to be identified. By floating that series of objects in an unknown liquid, the height of object above liquid was recorded. When plot the height of object above liquid (y-axis) versus the mass of floating object (x-axis), the absolute liquid density and the volume of a set of objects could be calculated from the slope and the intercept on y-axis, respectively. Pre-test and post-test were performed before and after using the teaching tool for a small group of 9 high school students in the preparation course before becoming undergraduate students. The improvement of post-test score indicated the understanding of buoyancy of the students. This simple teaching kit can be adapted for the teaching of buoyancy and linear equation for students in the secondary school.

Keywords: Density, Buoyant force, Archimedes' principle, Science kit

Introduction

Buoyancy is typically a day-to-day phenomenon. Ones can observe that when an object immersed or partially immersed in the liquid, it seems to be less weight than when it appears in air. Archimedes' principle, generally known, provided the information of that the force exerted upward on an immersed object equals to the weight of the liquid displaced by that object. This upward force is called buoyant force. According to the principle, the more of the weight of the liquid displaced by an object is, the higher the buoyant force becomes. The weight of liquid depends on the density and the volume of the liquid displaced by an object according to the following simple equation.

$$F_b = \rho V g \qquad (1)$$

F_b is buoyant force; ρ is the density of liquid; V is the volume of object immersed in the liquid and g is the gravitational acceleration. According to eq. (1), ones can assume that floating in the Dead Sea where the salt concentration is very high is probably easy than floating in the fresh water. In order to understand of the concept of the buoyant force, a simple experiment setup and the concept of the density were introduced to the students.

The density of an object is commonly defined as its mass divided by its volume or mass per unit volume. Density of matter is described as a scalar property with the SI unit of kg/m^3. Several methods using Archimedes' principle (Hughes, 2006; Kireš, 2007; Kulkarni, Kim and Kim, 2009) were demonstrated to identify the liquid densities. Hughes (Hughes, 2006) showed that the density could be calculated from the differences between the reading from the balance before and after the immersion of object divided by the volume of immersed object. The suspension, the balance and objects from immersion were necessary for the experiment.

In the article, Archimedes' principle in action (Kireš, 2007), Kireš used two beakers, one small and one large, to determine the density of unknown liquid. The small beaker was partially submerged in the large one which contained water. Kireš reported that the density of unknown liquid inside the small beaker could be calculated from the different volume between the volume of the water in the larger beaker before and after pouring liquid in smaller beaker divided by the volume of unknown liquid timed the density of water. Cantilever attached with the displacer was also used to find density of liquid in the earlier report (Kulkarni et al., 2009). The buoyancy force acting on the displacer when dipped into the liquid caused the deflection of cantilever. The deflection was then correlated to the density of liquid. Nevertheless, there were also methods which did not utilize Archimedes' principle. For example, Chattopadhyay provided a simple method to identify the liquid density using a metre rule with the principle of moment of forces only (Chattopadhyay, 2008).

In our case, the proposed simple technique to find liquid density is also based on Archimedes' principle which can be used in classroom especially at the level of junior or senior high school students. This method can provide the liquid density without knowing the mass or the volume of liquid. It can be designed to be a simple experiment which students can take data and plot their own graphs to find the intercept and slope for the calculation of liquid density and the volume of object.

Experimental

Proposed method

The volume of an object immersed in a liquid determines the buoyancy force. The force equals the weight of mentioned liquid replaced by the object. When the object partially immersed in a liquid, at equilibrium, the buoyancy force equals the weight of that object (figure 1) following eq. 2:

$$mg = \rho V_2 g \qquad (2)$$

m is mass of object; g is gravitational acceleration; ρ is the density of liquid and V_2 is the volume of object immersed in the liquid. If we know m and V_2, the density can be calculated. We can also derive the equation by replacing V_2 with $V - V_1$, then replace V_1 with Ah where h is the height of object above the water. A is the cross sectional area of object. For a simple calculation, the cylindrical object or the object with a uniform cross sectional area were evaluated. We would then obtain the following equations.

$$m = -\rho Ah + \rho V \qquad (3) \text{ or}$$

$$h = -\frac{1}{\rho A} m + \frac{V}{A} \qquad (4)$$

Both eq. 3 and eq. 4 make straight lines when plot h versus m or m versus h with the slopes of $-\rho A$ or $-\frac{1}{\rho A}$ and the intercepts of ρV or $\frac{V}{A}$, respectively.

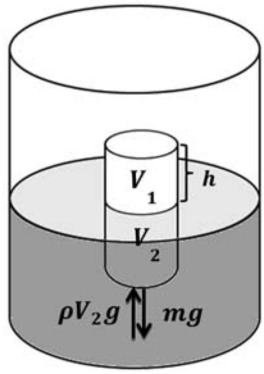

Figure 1 The schematic diagram of floating object for the determination of liquid density.

Experimentation

In order to obtain an accurate density, eq. 3 or eq. 4 can be utilized. In this experiment, a series of objects with known volume and mass were floated in the liquid. In our case, for simple experimentation, the small bottles were used as a series of known-volume objects. Mass of the bottle can be varied by adding powder, for example, sand into the bottles. Note that it is necessary to use fine substance to fill the bottle in order to maintain the equilibrium of the bottles when floated in a liquid. A cylindrical bottle is also a simple shape object that we could easily measure the volume of objects. A series of mass of object versus the height above the liquid were recorded and plotted.

In this experiment, 5 cylindrical plastic bottles (figure 2(a)) filled with sand at different mass of 80, 90, 100, 110 and 120 g were used. These bottles were slowly dropped in the liquid. The height of bottles above liquid could be easily recorded by students. The experiment setup is shown in figure 2(b). The liquid used in this experiment was prepared salt water with the density of 1.12 g/cm^3.

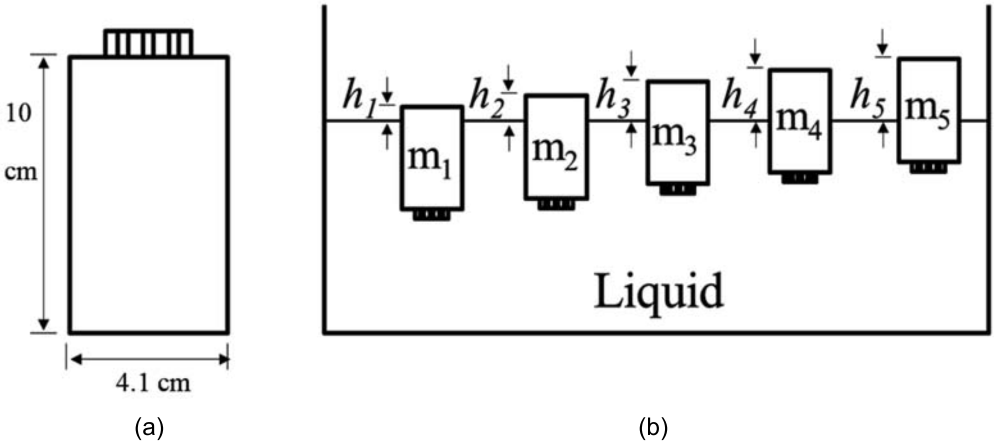

Figure 2 A series of mass of object versus the height above the liquid. (a) Diagram of the cylindrical bottles used in this experiment and (b) The schematic diagram of various floating objects for the determination of liquid density. The objects have the same shape and volume but different mass.

The teaching in action

The developed method and teaching technique were applied in the introductory for the fluid mechanics for a small group of senior high school students for the preparation before becoming undergraduate students. The students' prior knowledge should include the concept of density. The teaching divided into 3 stages, the pre-test session, the activity, and the post-test session. The pre-test question was "A cylindrical bottle with the diameter of 10 cm contained some candy. The distance from the bottom of the bottle in the water to the surface of water is 20 cm. Calculate the mass of the candy inside the bottle?" The students were then graded on the concept of buoyant force by checking if the students remember and understand eq. 2 or not. After the pre-test, teacher then provided the definition of buoyant force and drew force diagram to explain the relationship between mass of the bottle and the volume of the bottle immersed in the water.

During the teaching, teacher proposed example experiments to determine the density of unknown liquid by buoyant force. The science kit consisting of 5 cylindrical plastic bottles with the mass of 80, 90, 100, 110 and 120 g, the unknown liquid in a container, a ruler and a graph sheet was provided to the students. In our case, the unknown liquid was prepared salt water. The instruction was to find the density of the given unknown liquid using the provided equipment. During the experiment, students had a chance to brain-storm and had hand-on experience. Students might try floating the bottles in the given liquid and measures the height of bottles immersed in the liquid or over the liquid. During the experiment, teacher asked guiding question, for example, "why do we have to measure the height of the bottles?", "Are there any differences between measuring height of the bottles immersed in the liquid or over the liquid?", "How can you use 5 given bottles instead of 1 bottle to better estimate the liquid density?" and "How can you use the provided graph sheet?".

When students started to have ideas about how to do the experiment, teacher then started to develop eq.3 or eq. 4 from eq. 2. After having eq. 4, teacher showed that height (h) and mass (m) had a linear relation by comparing to a conventional simple linear equation, y = mx + c. At this point forward, students should be able to continue the experiment on their own. At the end, teacher mentioned roughly about significant figures and suitable number of trials to obtain reason-able results.

To check their understanding, the next stage was another activity session which students needed to answer the question, "How many coins do you want to add in the bottles to make the bottle to sink at a given level?" In the activities, the apparatus consisted of one plastic bottle and numbers of coins. During the activity, teacher encouraged the students to discuss and searching for the necessary data on their own.

For the post-test question, students were asked to answer the question of "If a cylindrical bottle with the diameter of 10 cm and the height of 10 cm just completely immersed in the water, what is its mass?" The post-test was not conducted at the end of the activity, but it was conducted 3 day after the activity to observe the retention of the students' understanding.

Results and discussion

Analysis of the teaching tool

Bottles filled with sand whose mass (m) varied from 80 g to 120 g were floated in the liquid, and the height (h) of bottles above the liquid was recorded for 5 times. The recorded heights and the plot of h vs. m are shown in Table1 and figure 3, respectively. We can obtain the equation of the straight line (eq. 5).

$$y = -0.0661x + 9.954 \qquad (5)$$

When compare this with eq. 4, we obtain the following equations.

Table 1 Height of the bottle above an unknown liquid when the bottle partially immersed in the liquid versus mass of the bottle.

Mass of the bottle (g)	Height of the bottle above an unknown liquid (cm)					
	no. 1	no. 2	no. 3	no. 4	no. 5	average
80	4.70	4.75	4.80	4.75	4.65	4.73
90	4.05	4.00	4.00	4.10	4.00	4.03
100	3.30	3.25	3.15	3.20	3.20	3.22
110	2.65	2.60	2.55	2.60	2.60	2.60
120	2.15	2.05	2.10	2.20	2.20	2.14

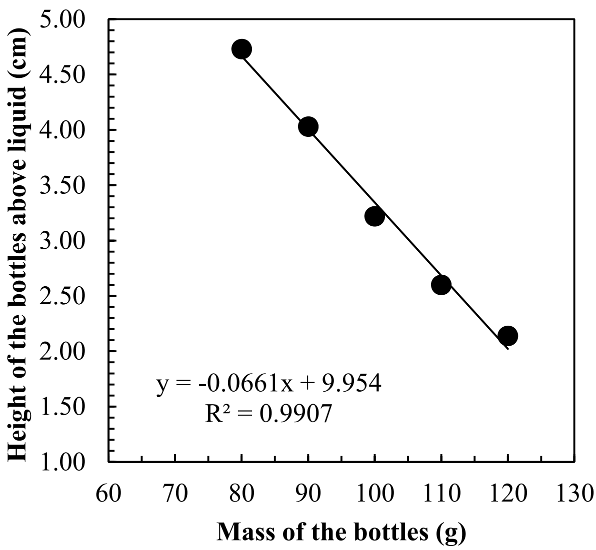

Figure 3 The relationship between the heights above the liquid of the cylindrical bottles when floating with their mass.

$$\text{slope} = -\frac{1}{\rho A} = -0.0661 \qquad (6)$$

$$\text{intercept} = \frac{V}{A} = 9.954 \qquad (7)$$

Area (A) of the bottom of bottle is 13.2 cm^2. Therefore, ρ is 1.15 g/cm^3.

The obtained density is close to the density of the prepared salt water. However, the main sources of error were the measurement of the height of bottle above the liquid and also the shape of bottles which is not real cylindrical.

The student learning outcome

The learning outcome of the students was determined using pre-test and post-test scores of the students before and after the activities. Due to the simplicity of question, the questions were graded only into 3 levels: Correct, Partial, Incorrect. A small group of 9 students was tested with this method. For the pre-test, it was found that only 3 students can provide the correct answer, 1 can partially provide the concept of buoyancy, while the rest cannot answer the question. After the activities, 8 out of 9 students can answer question correctly. The suggested activity was a good method to improve the students' understanding.

Conclusion

A simple technique to measure liquid density was developed. A series of known-volume objects with various masses can be used to measure a liquid density. By floating that series of objects in an unknown liquid, the height of object above the liquid can be measured. A simple relationship between mass of objects and their height above the liquid can provide us the liquid density and also the total volume of object. Besides the concept of density, one of the useful parts of this method is that students can also learn about taking data, finding relationship between data and extracting useful information.

Acknowledgment

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